Neutron Interferometry

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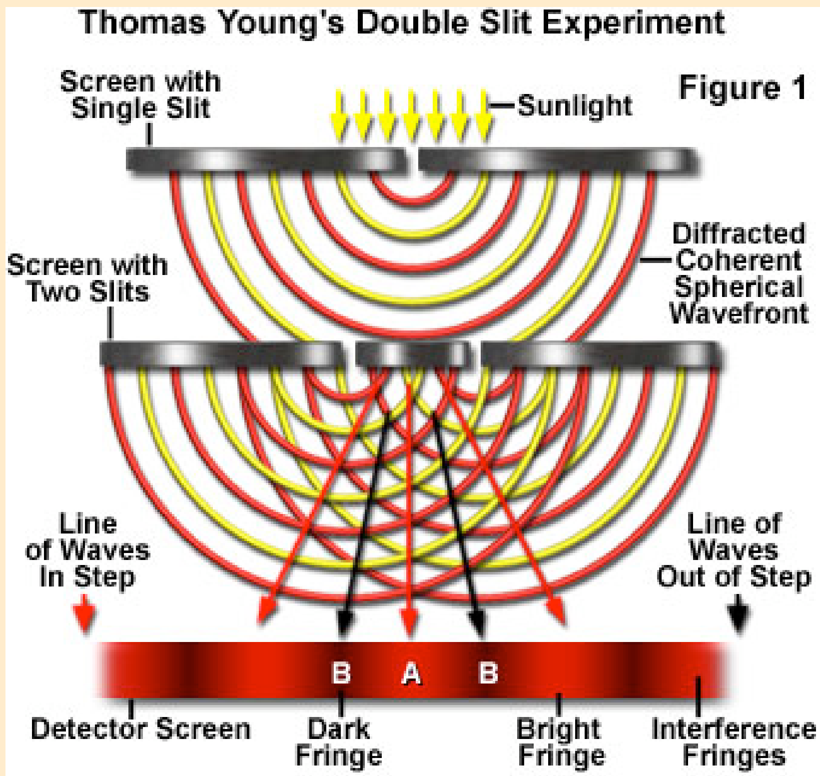
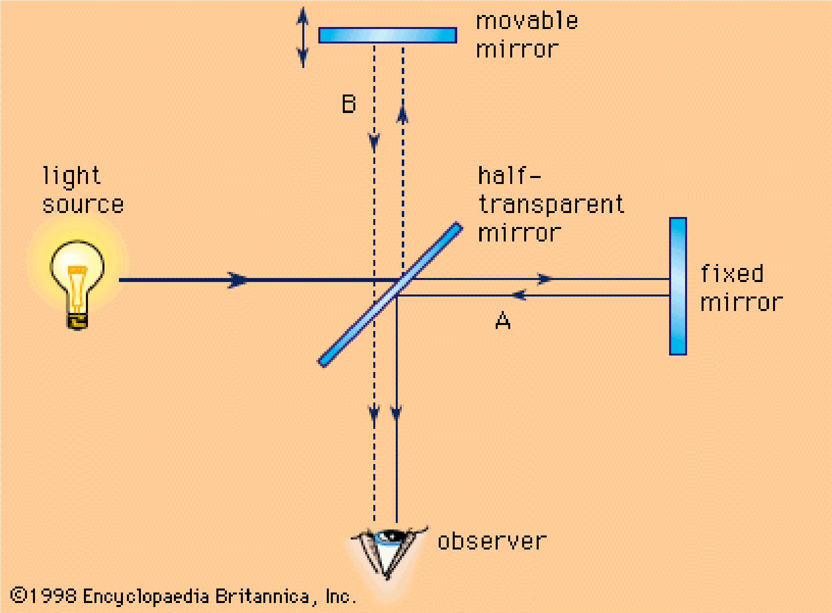
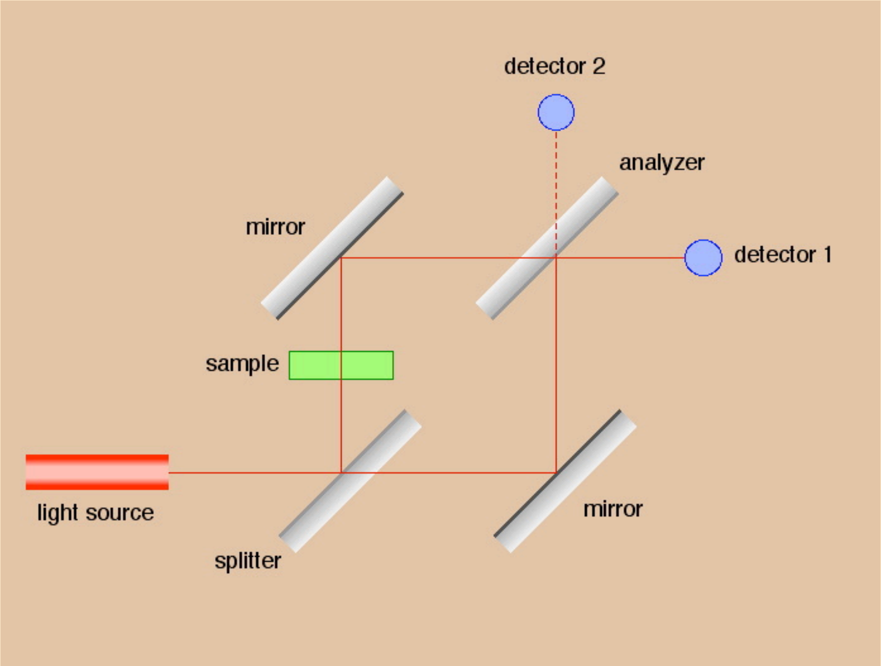
What is an interferometer?

An interferometer is a device that measures the relative phase between coherent waves by detecting the interference pattern.



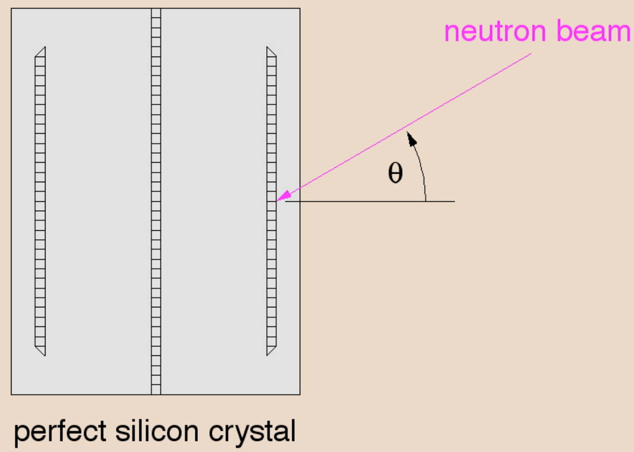
Michelson Interferometer





Mach-Zender Interferometer

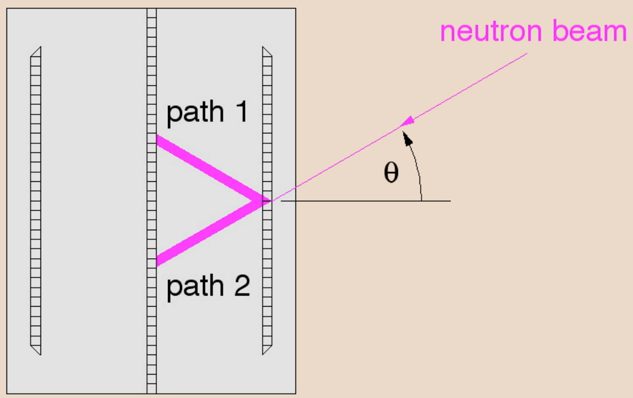
Perfect Crystal LLL Neutron Interferometer



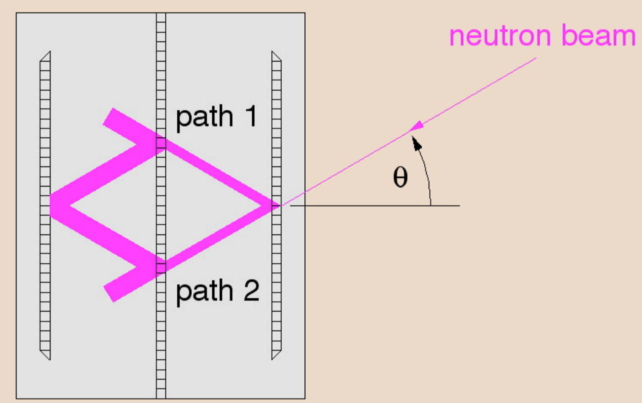
Bragg condition: $n\lambda = 2d\sin\theta$

d = lattice spacing

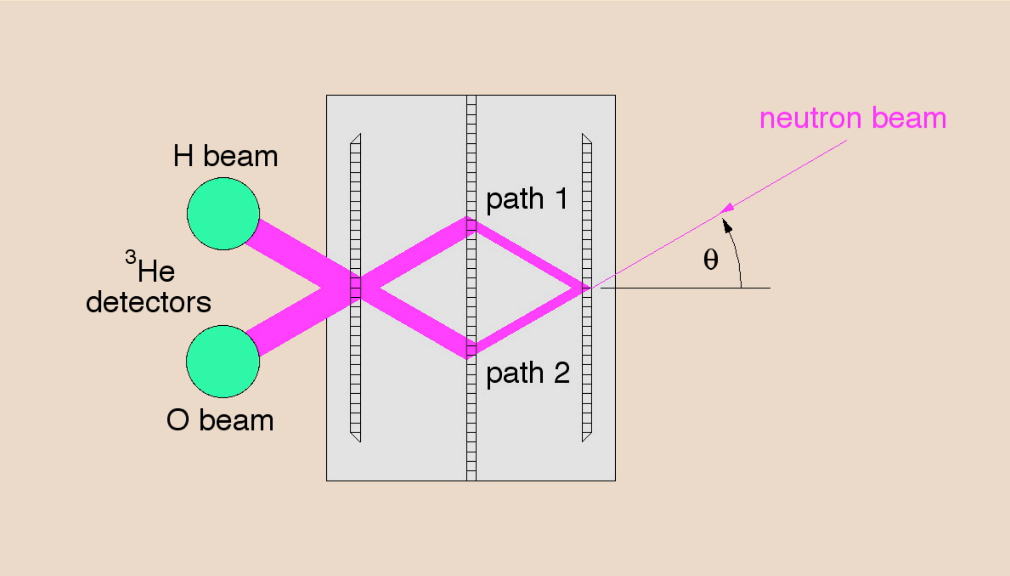
Perfect Crystal LLL Neutron Interferometer



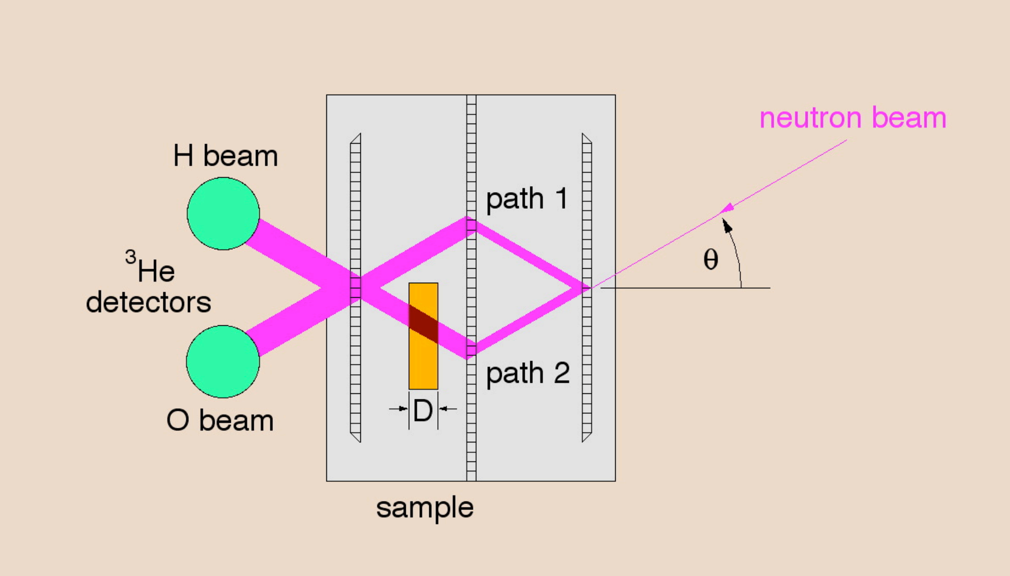
Perfect Crystal LLL Neutron Interferometer



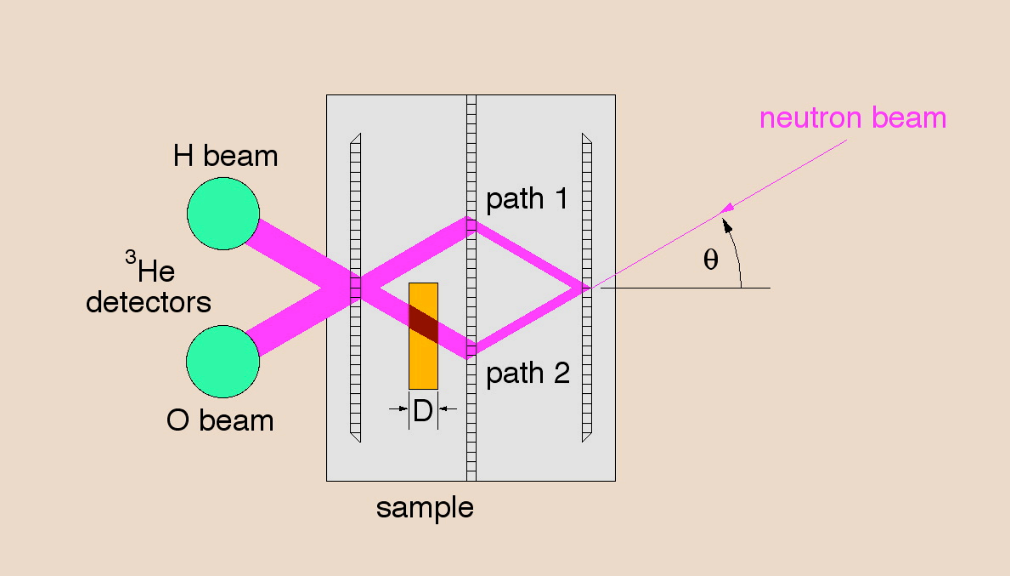
Perfect Crystal LLL Neutron Interferometer

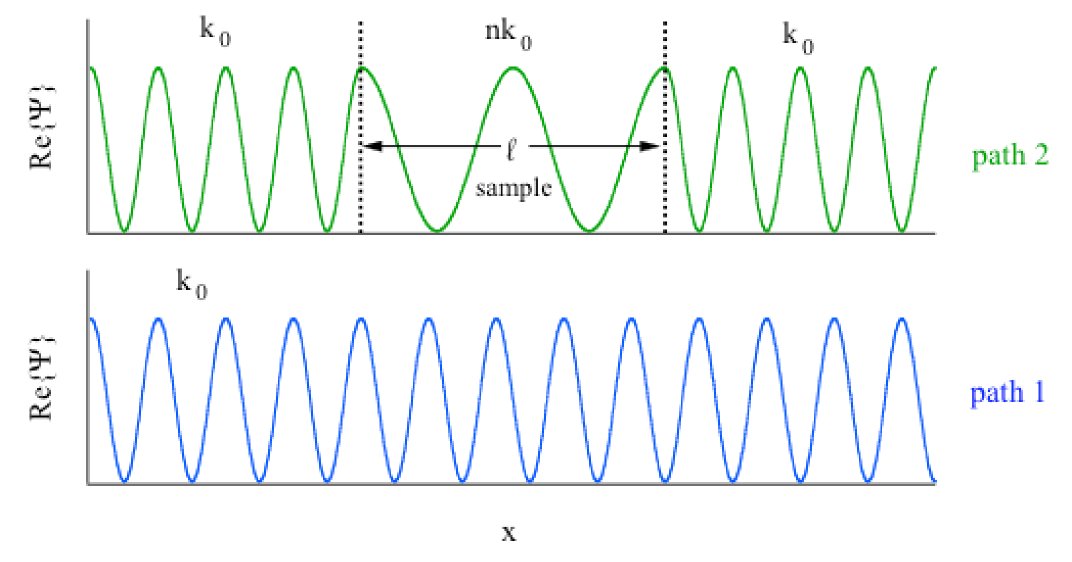


Nuclear Phase Shift



Nuclear Phase Shift



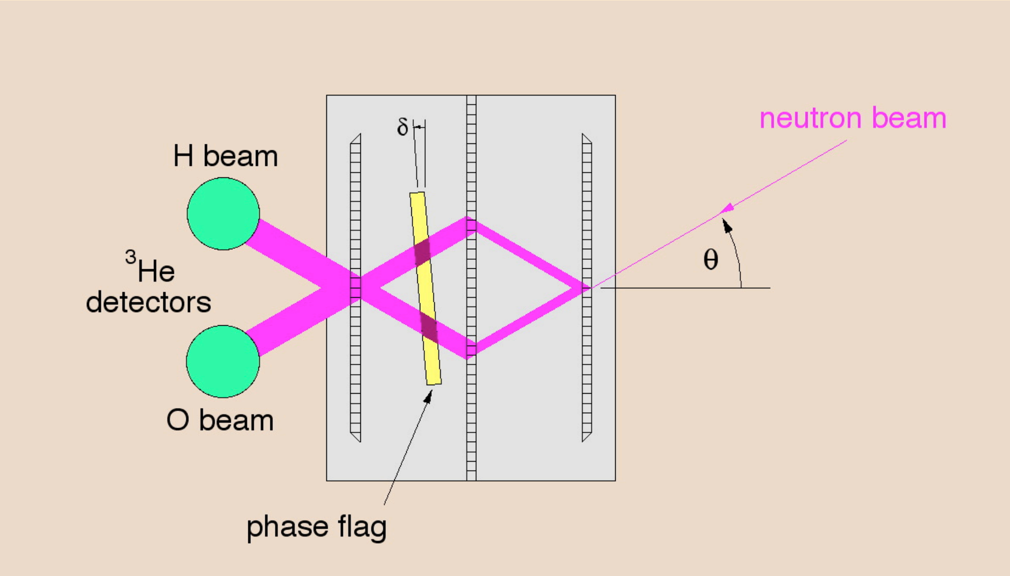


index of refraction: $n = 1 - \dfrac{Nb\lambda^2}{2\pi}$

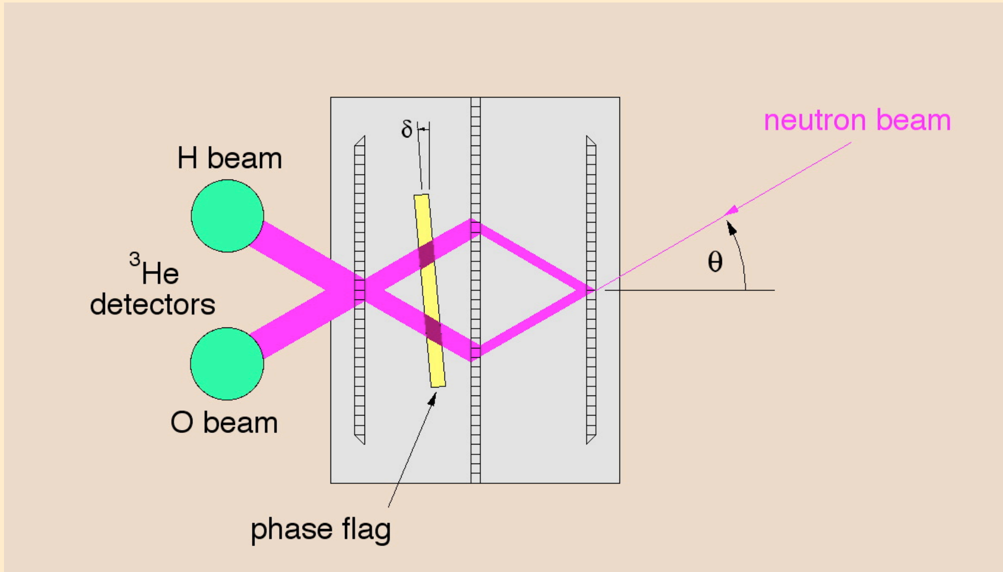
relative phase shift:

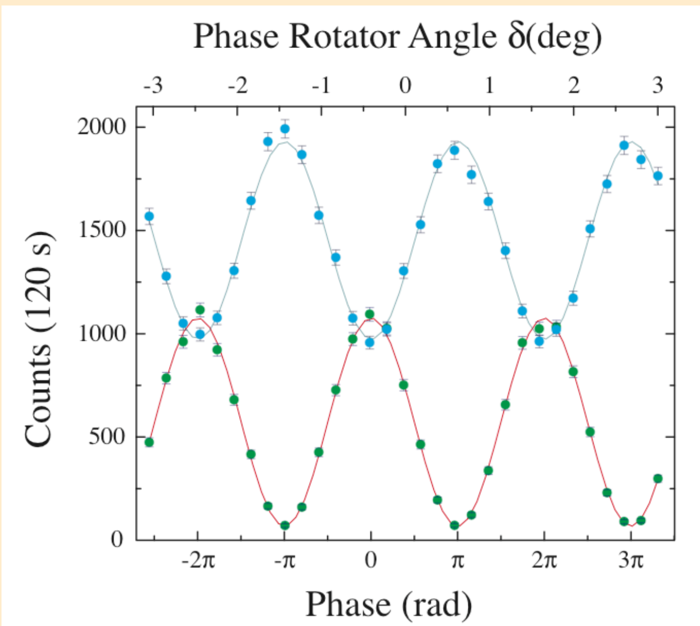
$$\Delta\chi = k_0\ell - nk_0\ell = Nb\lambda \frac{D}{\cos\theta}$$

Interferogram



Interferogram



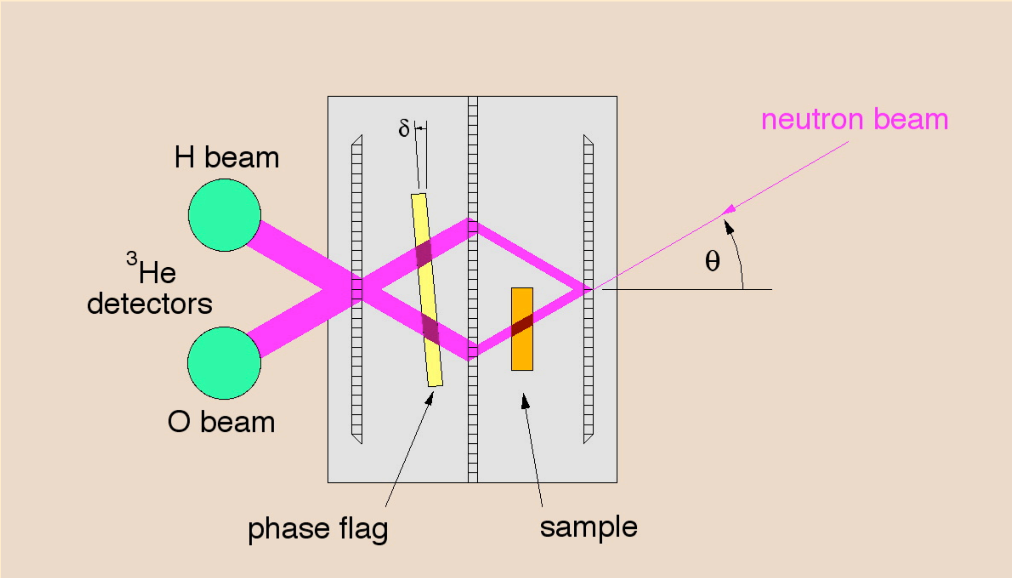


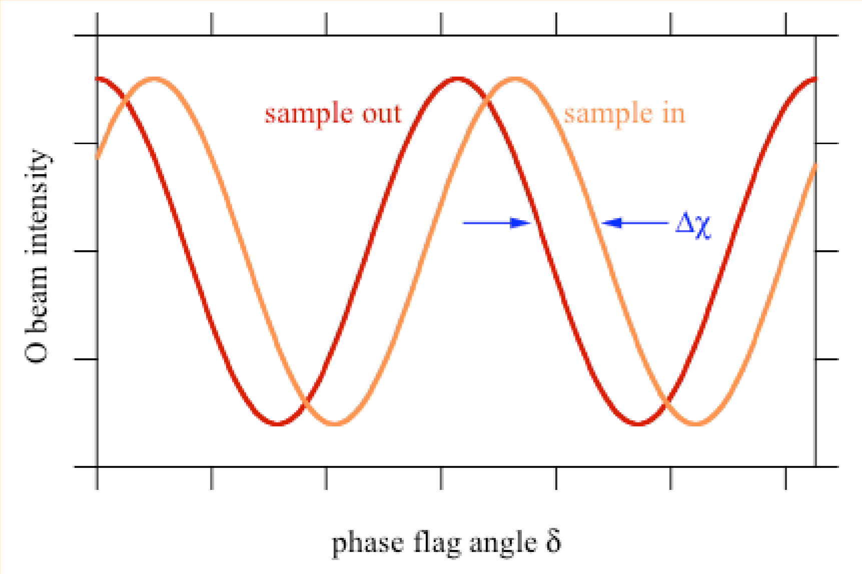
O beam: $I_O = A\left[1 + f\cos(\chi_2 - \chi_1)\right]$

H beam: $I_H = B - Af\cos(\chi_2 - \chi_1)$

contrast $f = \dfrac{C_{max} - C_{min}}{C_{max} + C_{min}}$ (O-beam)

Precision Phase Shift Measurement





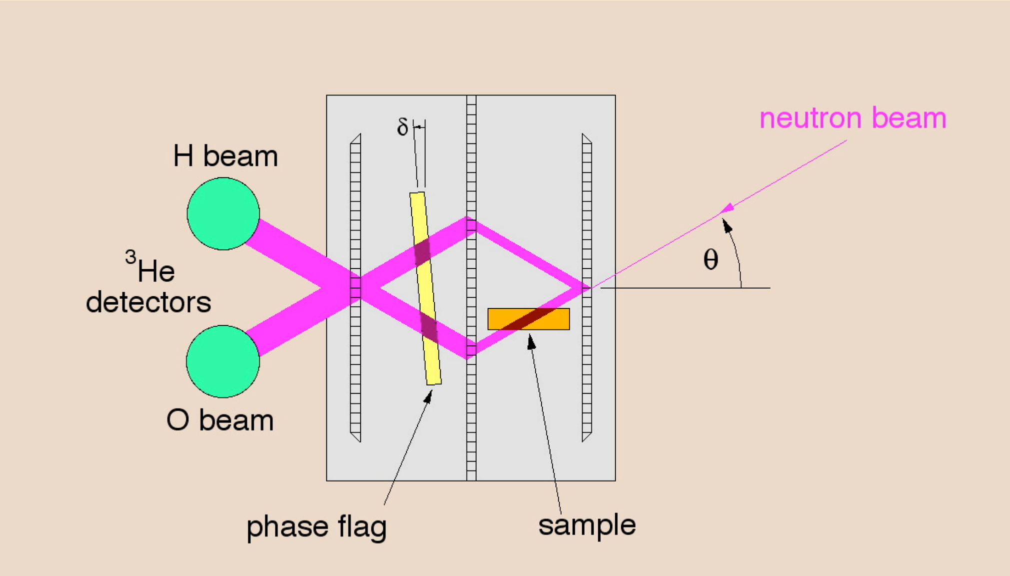
$$\Delta\chi = Nb\lambda \frac{D}{\cos\theta}$$

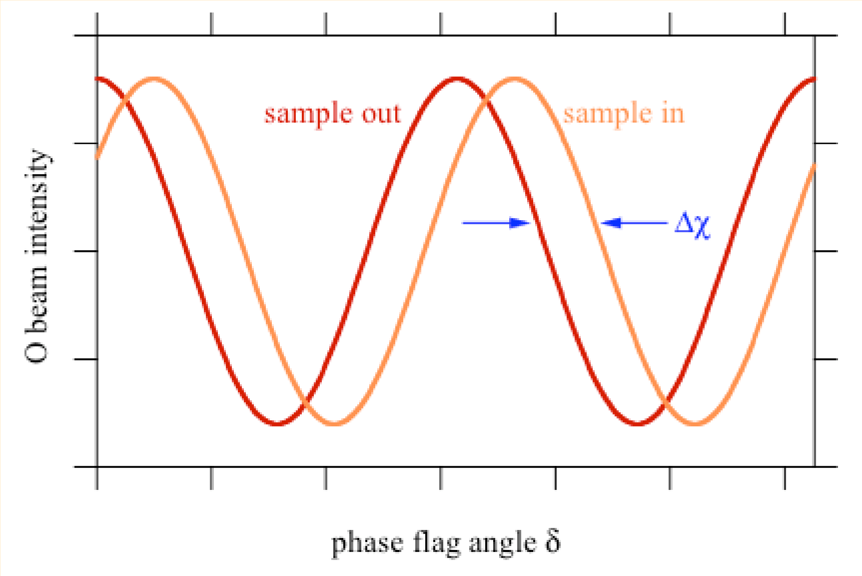
Example:

aluminum sample, λ = 2.70 Å, ⟨111⟩ reflection

$D = 100\ \mu\text{m} \Rightarrow \Delta\chi = 2\pi$

Non-Dispersive Geometry



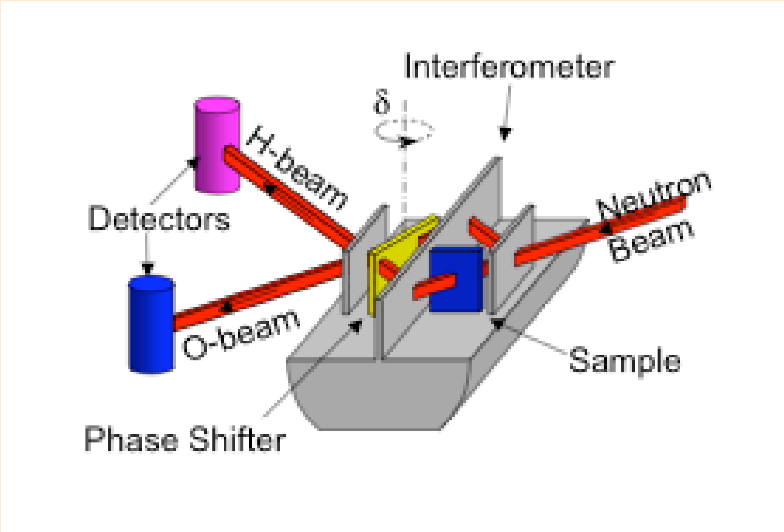


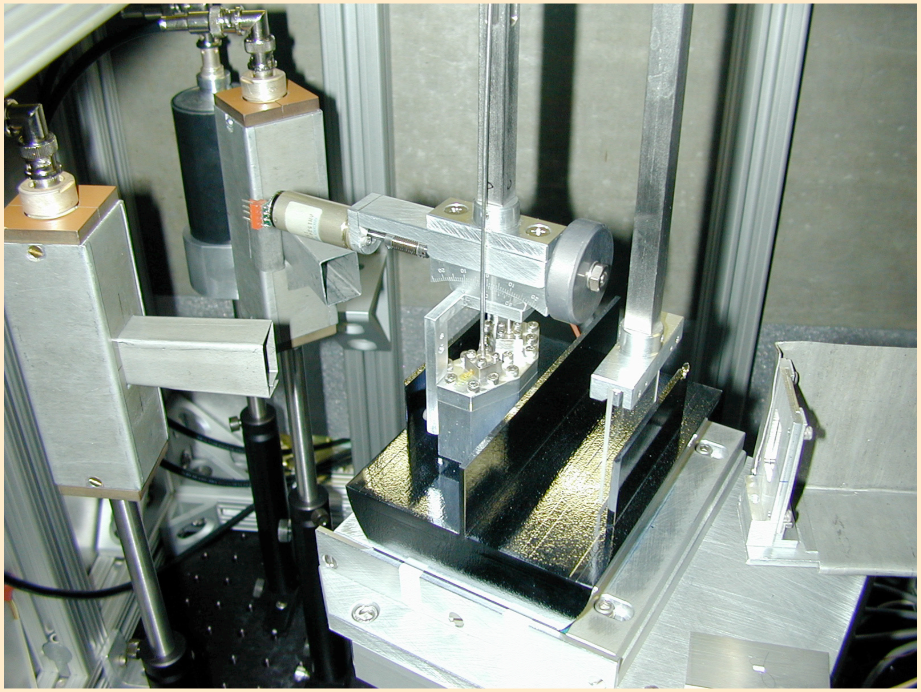
path length $\ell = \dfrac{D}{\sin\theta}$

$$\Delta\chi = 2NbdD$$

independent of λ

Perfect Crystal LLL Neutron Interferometer





PHYSICAL REVIEW A VOLUME 58, NUMBER 2 AUGUST 1998

Precision neutron-interferometric measurement of the coherent neutron-scattering length in silicon

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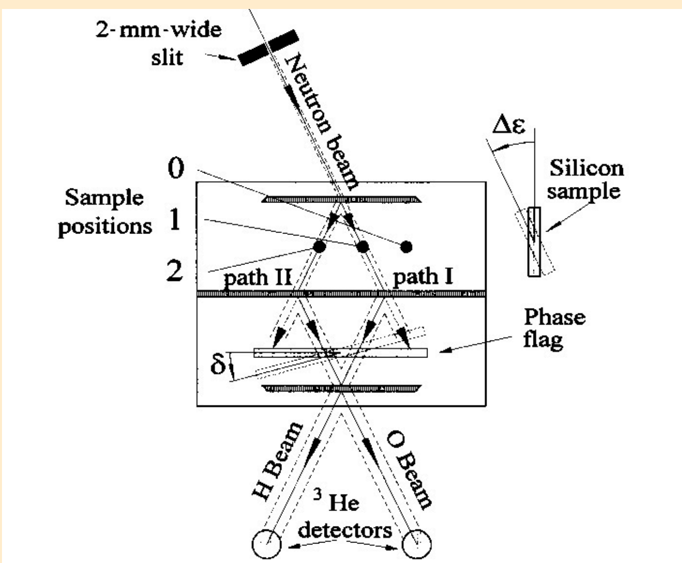
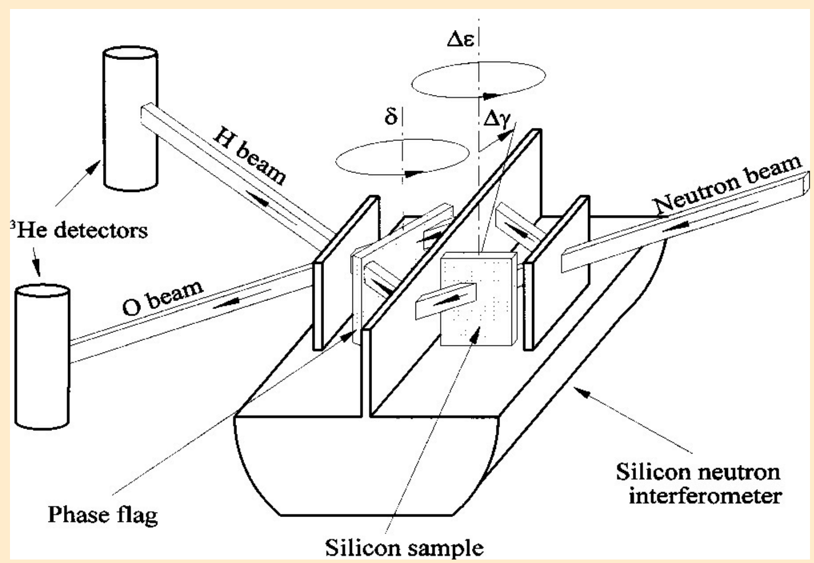
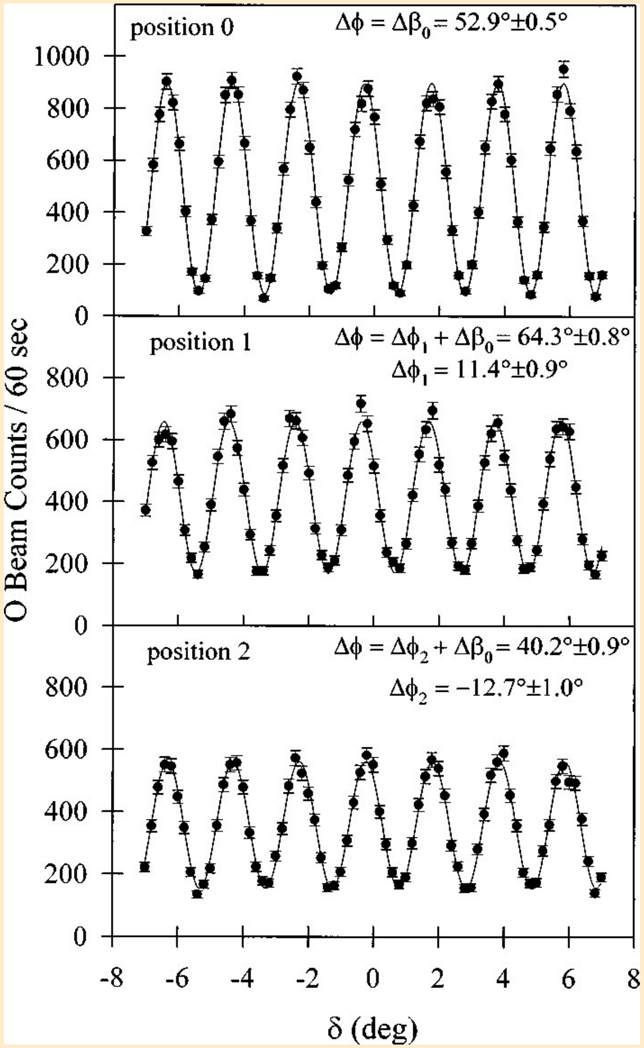
(Received 15 August 1997)

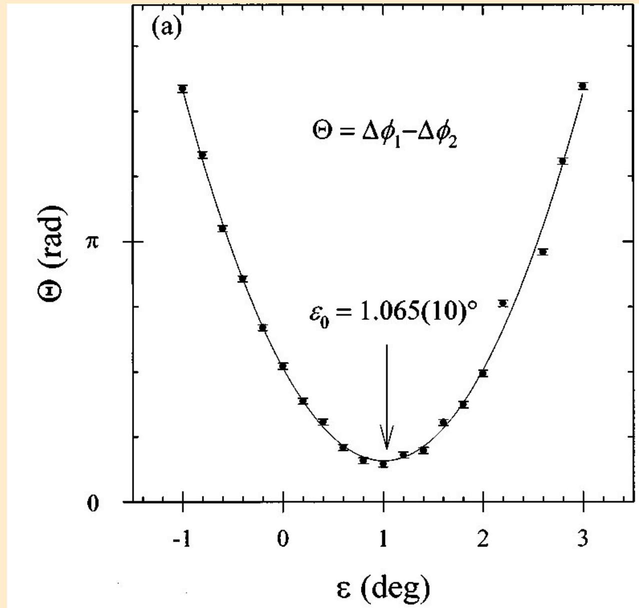
The neutron-interferometry (NI) technique provides a precise and direct way to measure the bound, coherent scattering lengths b of low-energy neutrons in solids, liquids, or gases. The potential accuracy of NI to measure b has not been fully realized in past experiments, due to systematic sources of error. We have used a method which eliminates two of the main sources of error to measure the scattering length of silicon with a relative standard uncertainty of 0.005%. The resulting value, $b = 4.1507(2)$ fm, is in agreement with the current accepted value, but has an uncertainty five times smaller. [S1050-2947(98)04808-2]

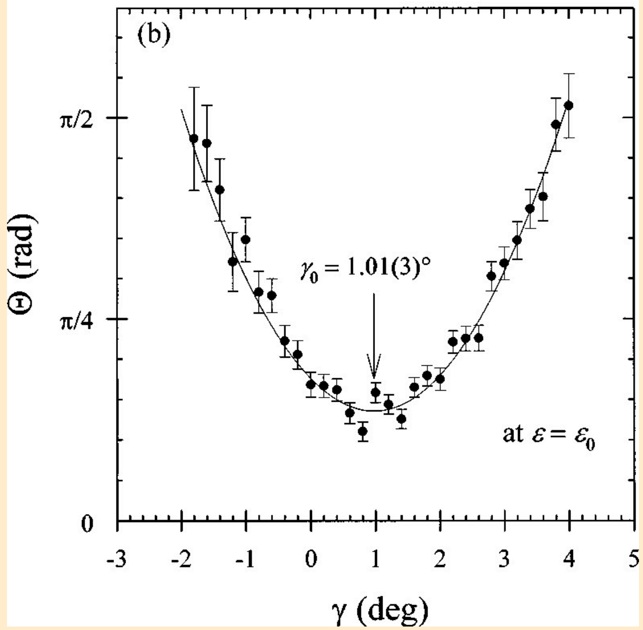
PACS number(s): 03.75.Dg, 07.60.Ly, 61.12.−q







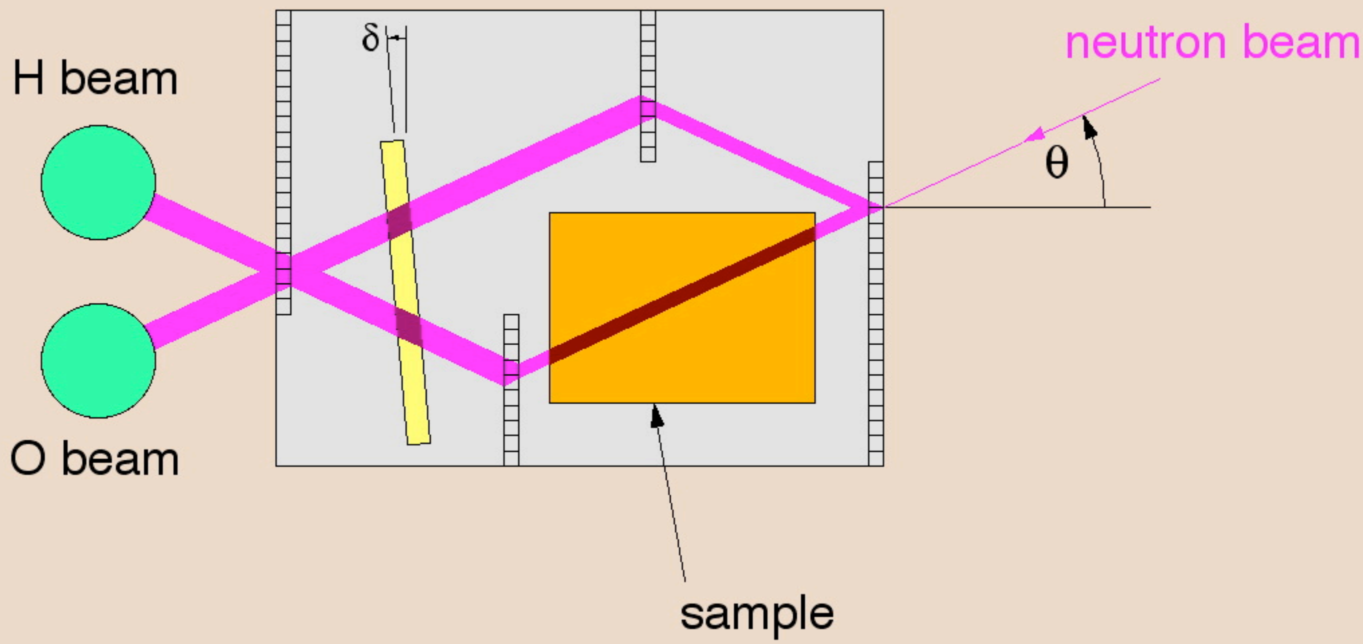




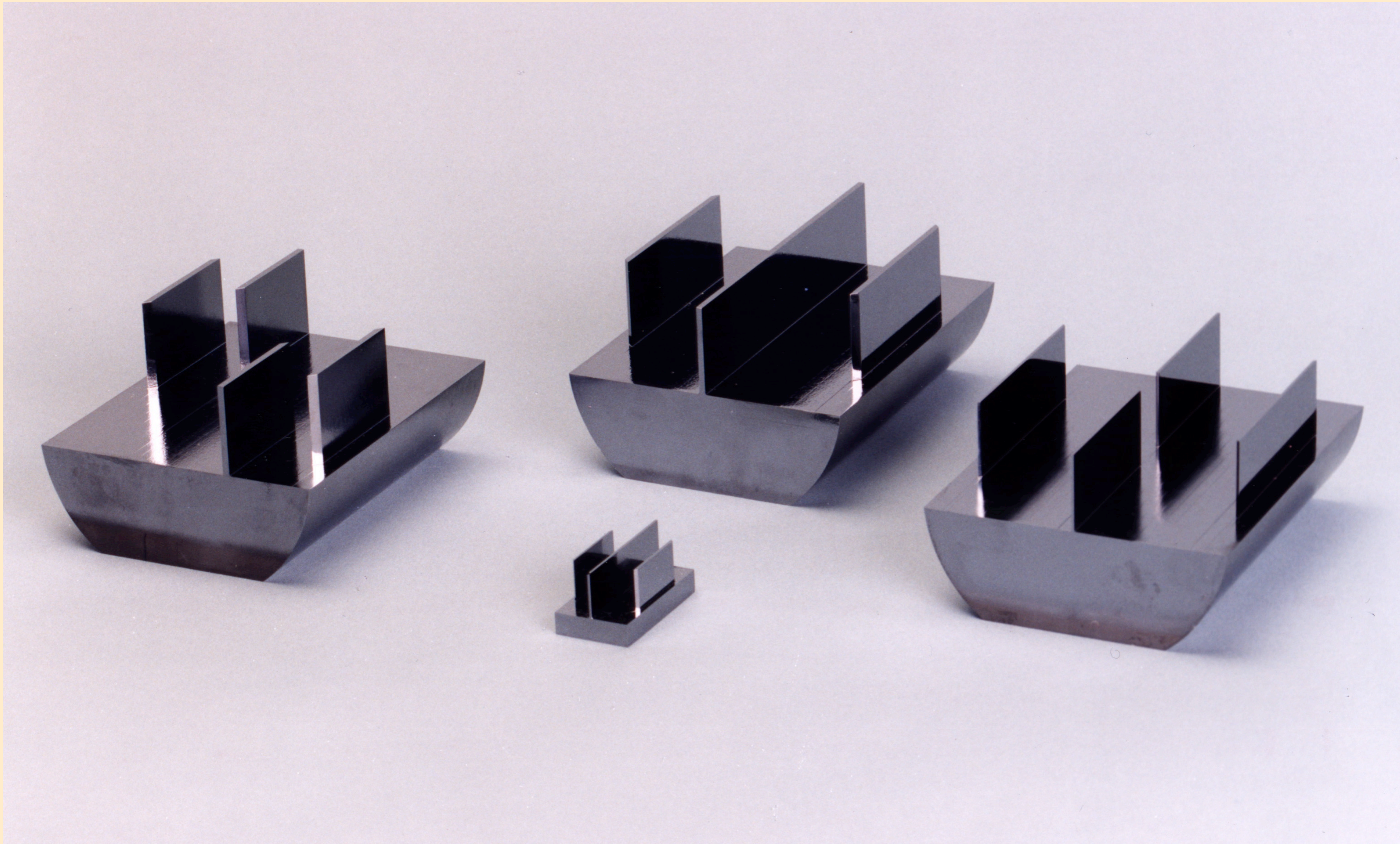
net phase shift: $\Theta\left(\varepsilon_0, \gamma_0\right) = 248\pi + 0.455(7)$ radians

$b_{\mathrm{coh}} = 4.15041(21)$ fm

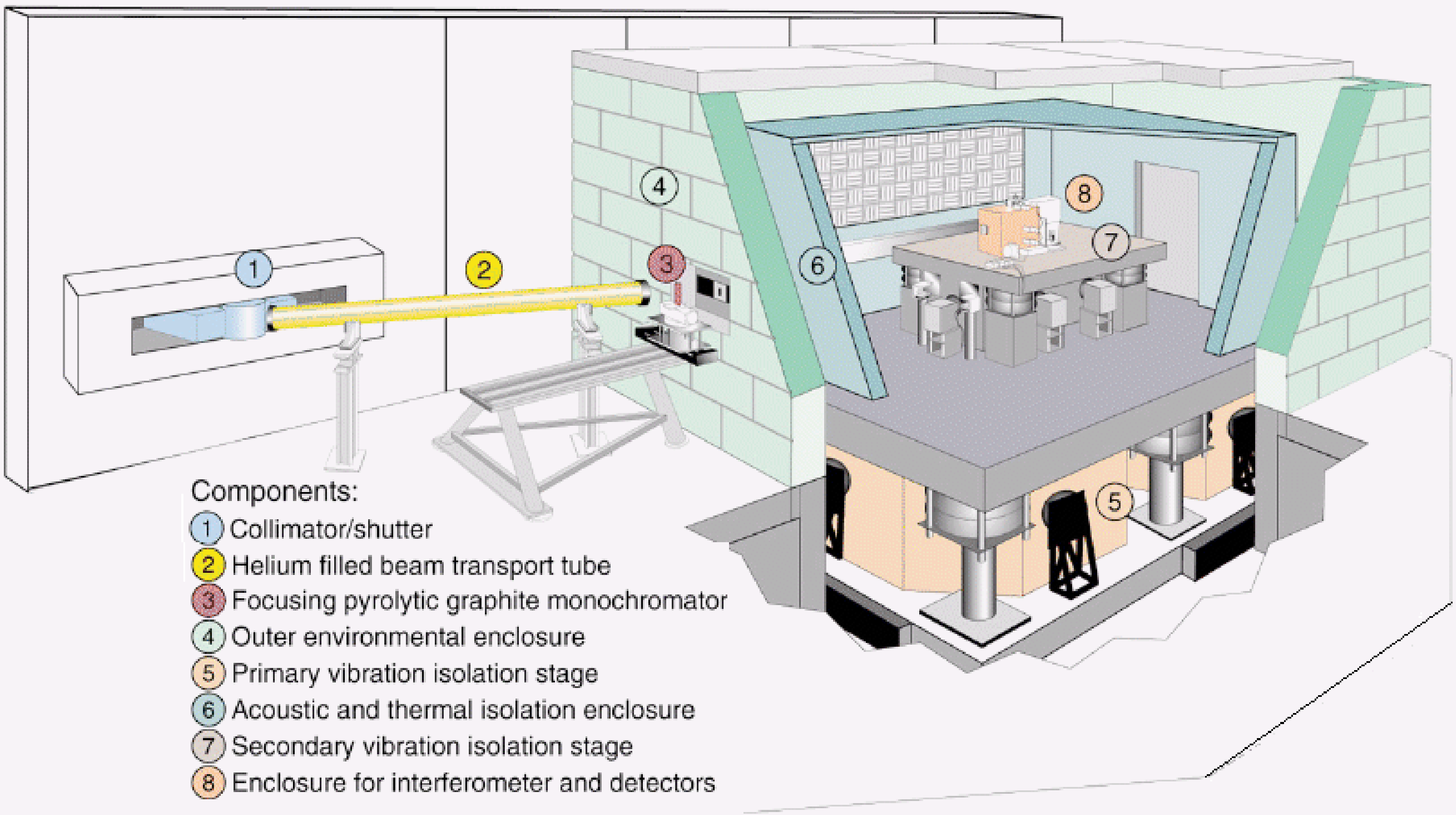
Skew-Symmetric Neutron Interferometer



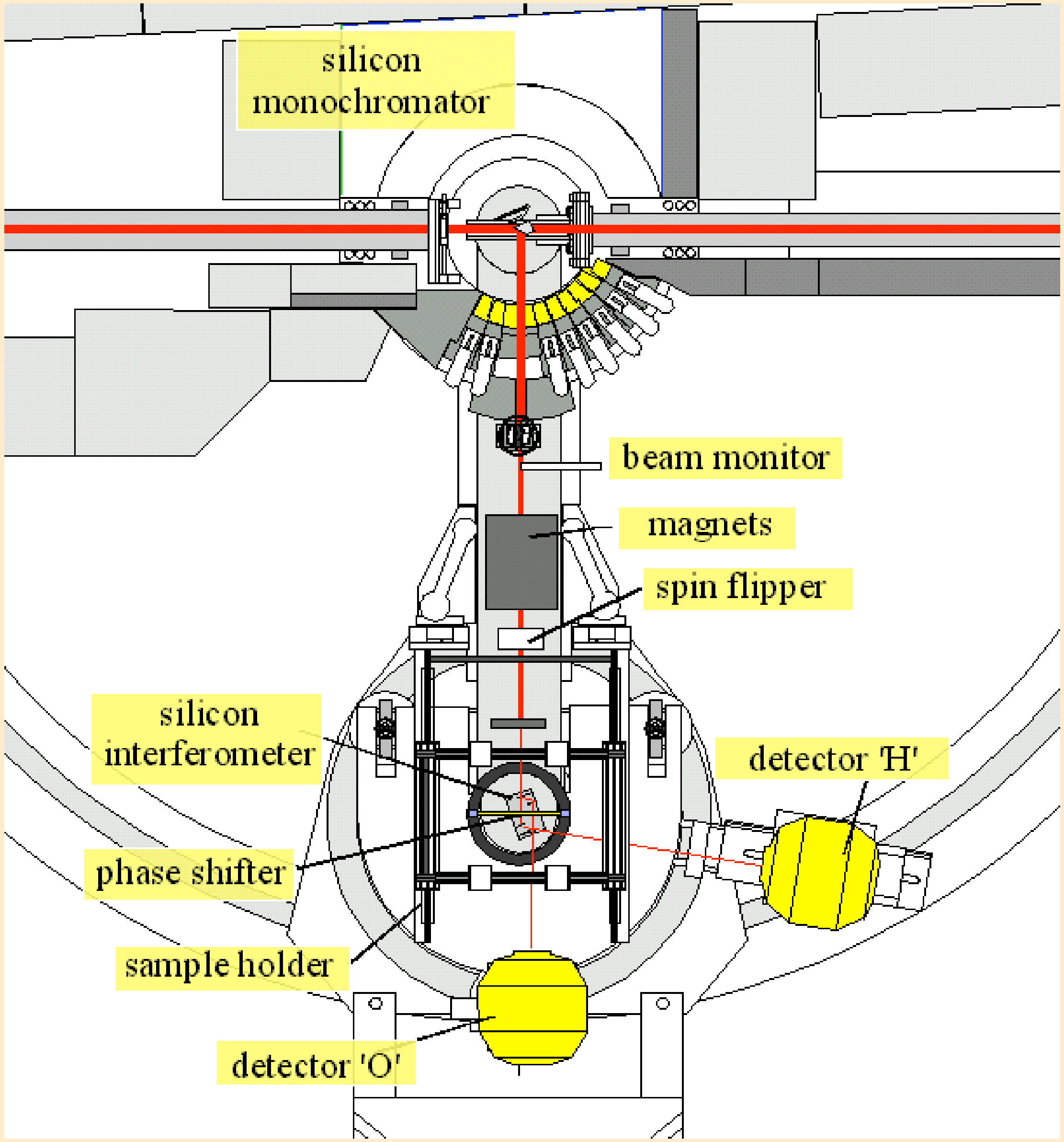
NIST perfect crystal silicon interferometers



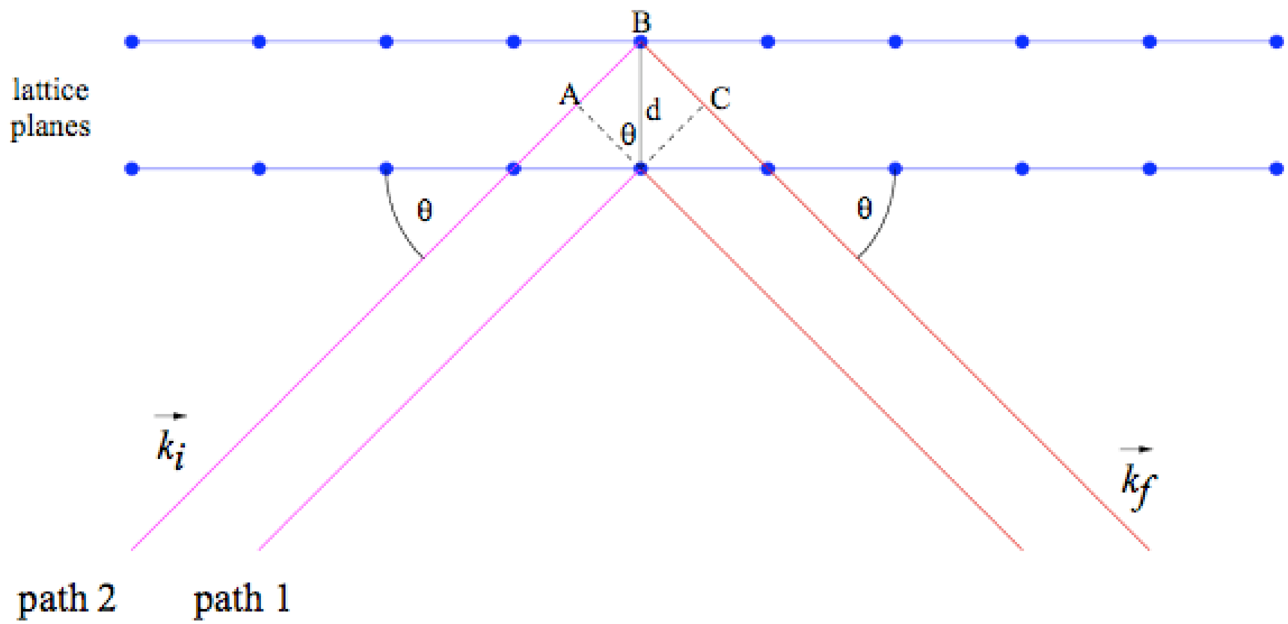
Neutron Interferometer and Optics Facility



S18 Neutron Interferometer at the Institut Laue-Langevin



"Geometric" Bragg Reflection

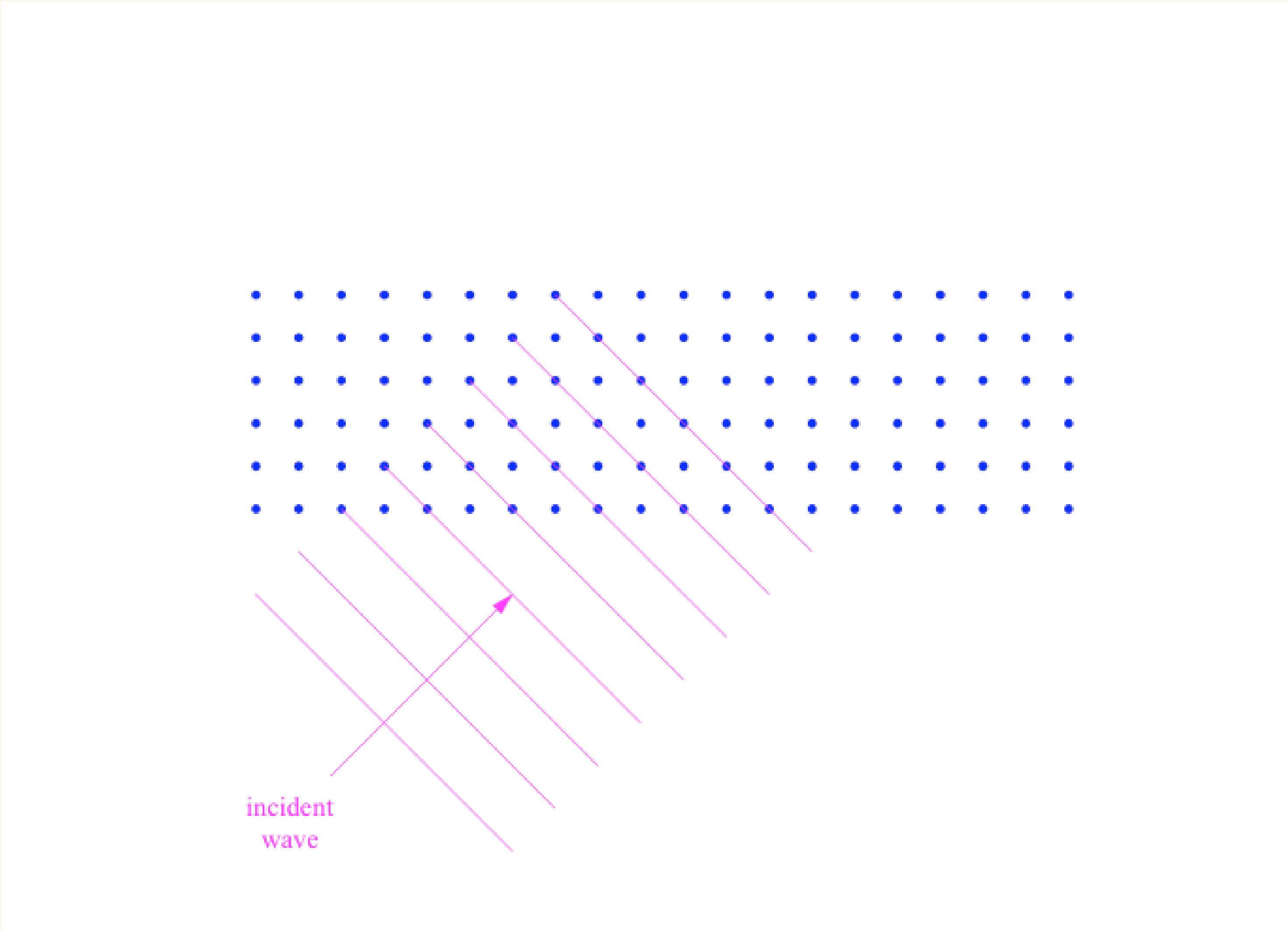


path 2 travels addition distance $\overline{AB} + \overline{BC} = 2d \sin\theta$

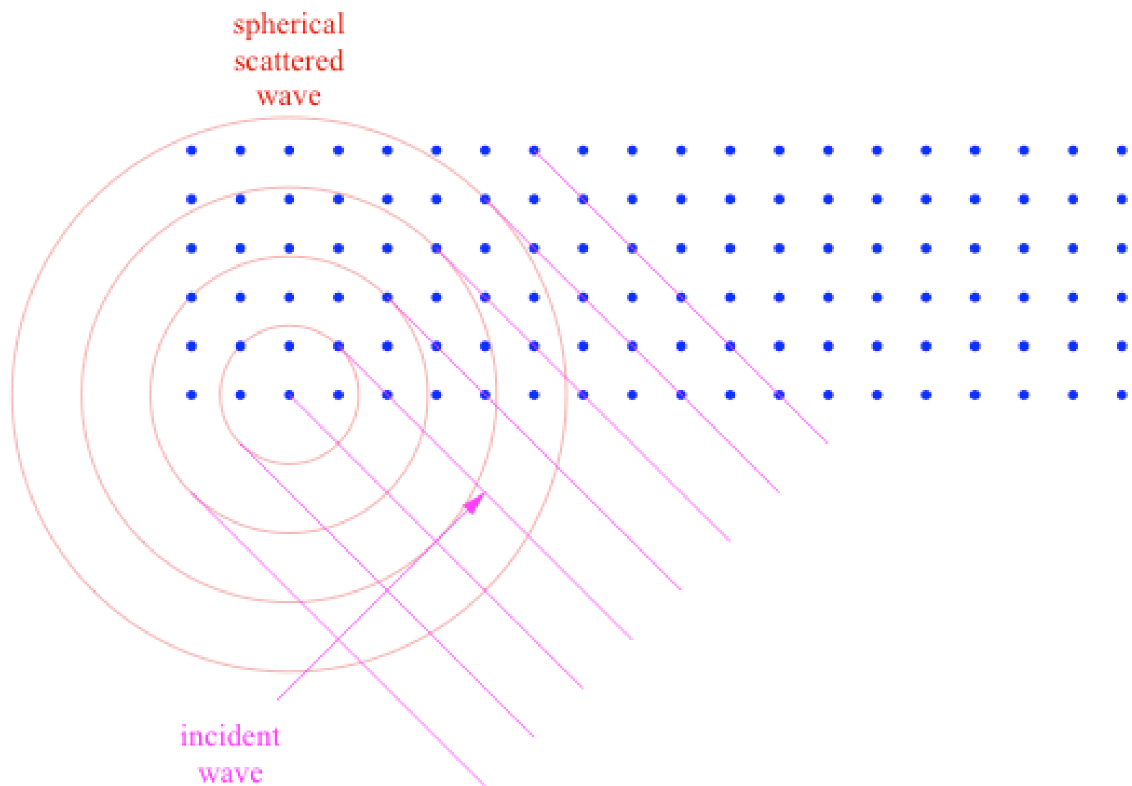
relative phase shift: $\Delta\phi = \dfrac{2d \sin\theta}{\lambda} \times 2\pi$

condition for constructive interference: $n\lambda = 2d \sin\theta$ (Bragg's Law)

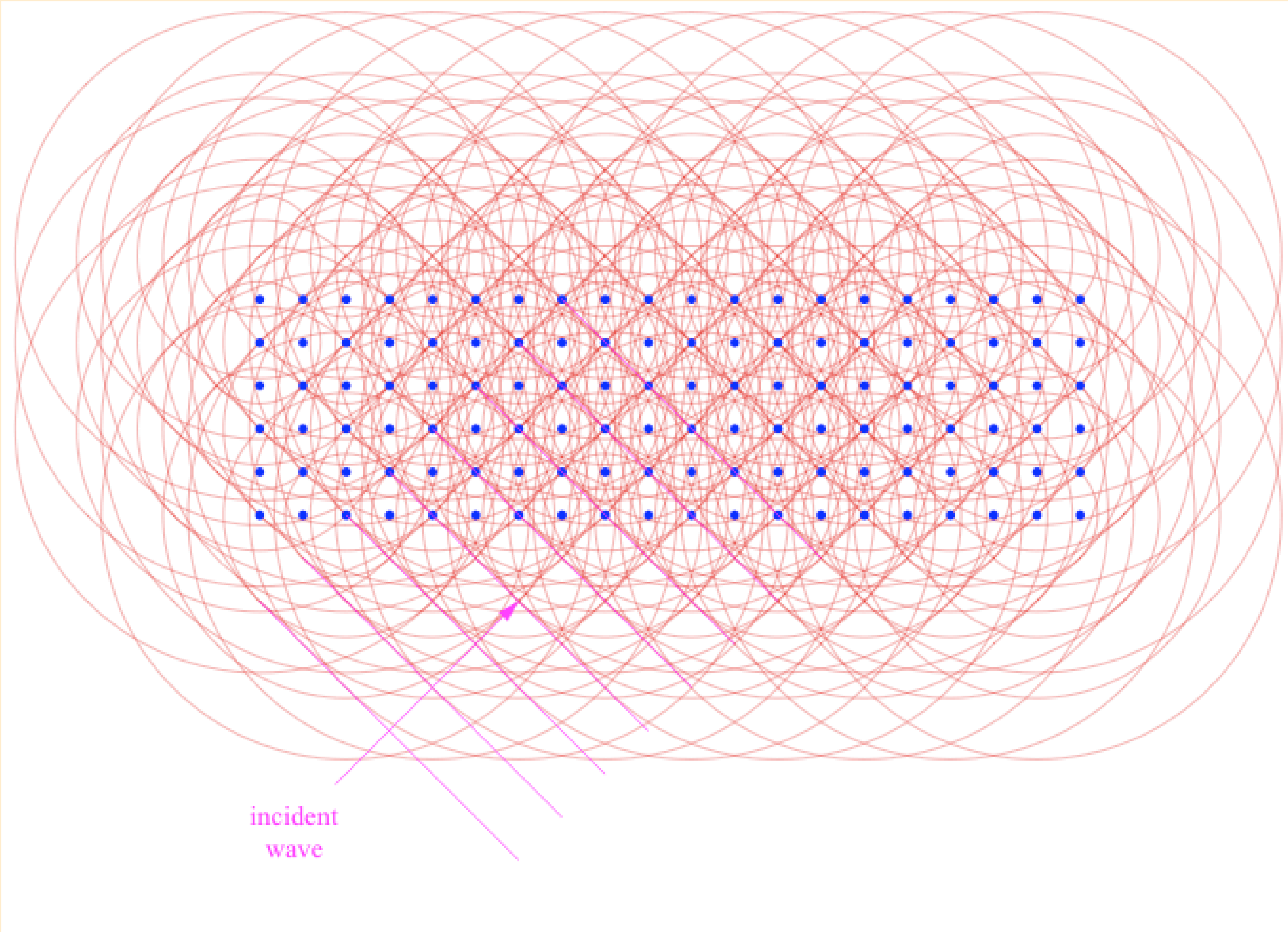
Kinematic Bragg Diffraction



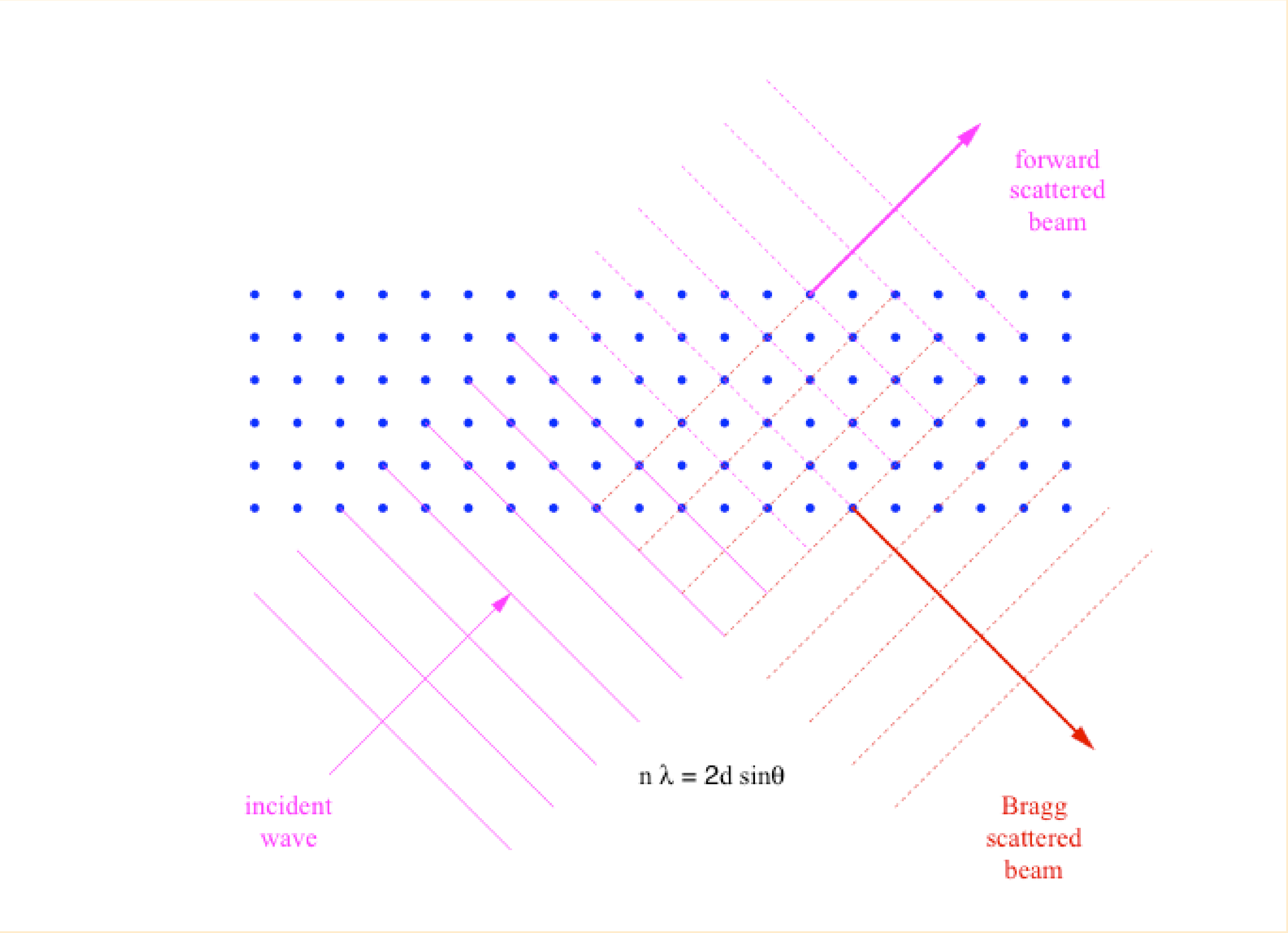
Kinematic Bragg Diffraction



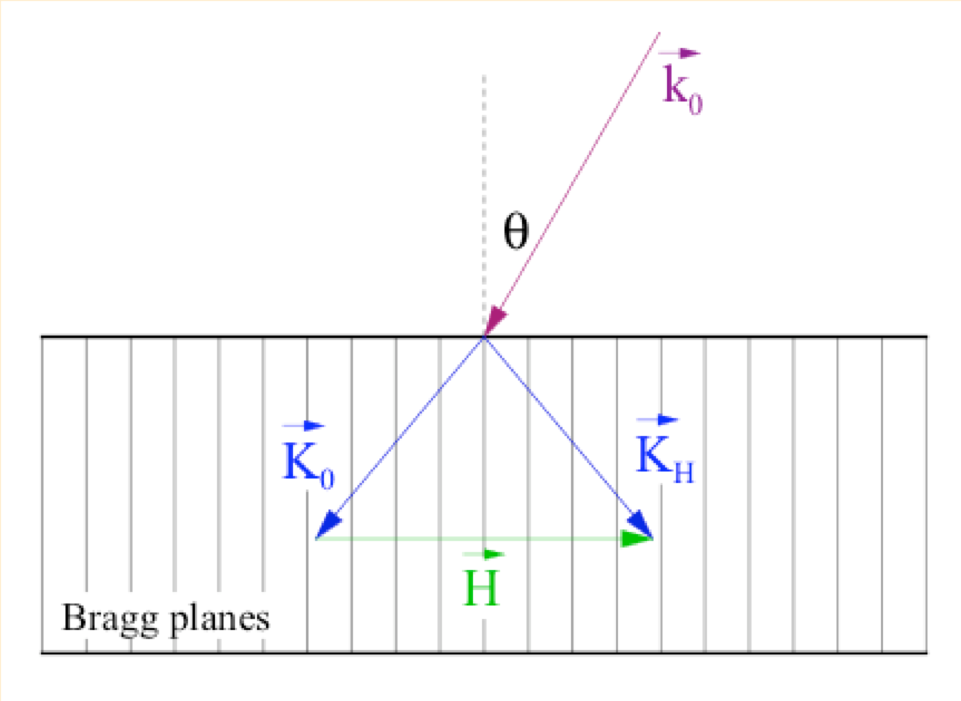
Kinematic Bragg Diffraction



Kinematic Bragg Diffraction



Dynamical Diffraction Theory



$\vec{H}$ = Bragg vector

$$\left|\vec{H}\right| = \frac{2\pi n}{d}$$

$\vec{K}_0$ = internal forward scattered wave

$\vec{K}_H$ = external forward scattered wave

Bragg condition:

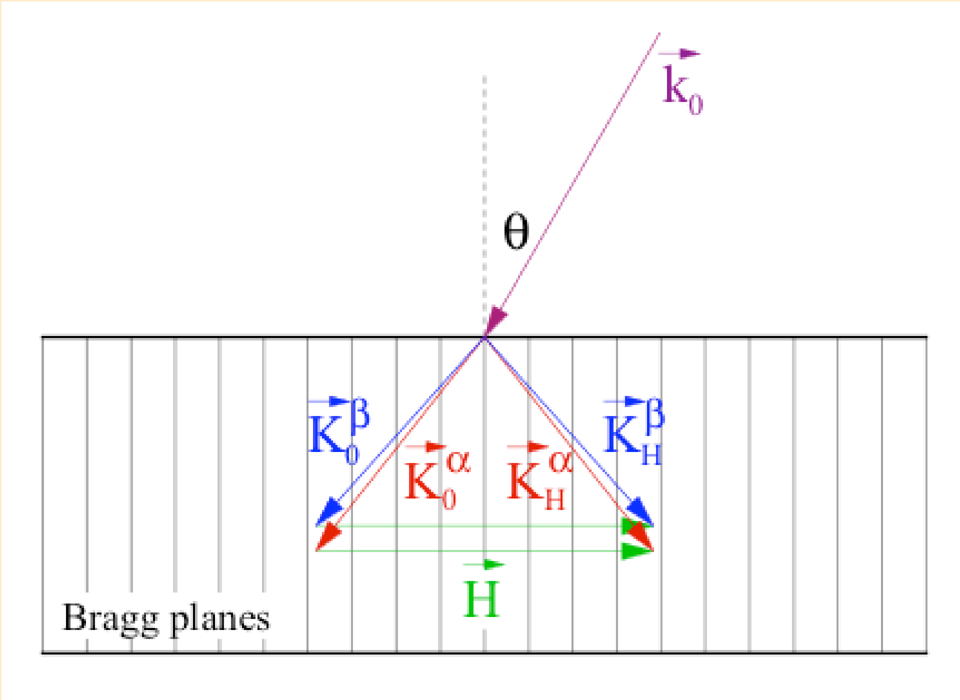
$$\vec{K}_H - \vec{K}_0 = \vec{H}$$

Solve Schrödinger Eqn. inside crystal:

$$\left(\nabla^2 + k_0^2\right)\Psi(\vec{r}) = v(\vec{r})\Psi(\vec{r})$$

$$\text{with} \quad v(\vec{r}) = 4\pi\sum_i b_i\delta\left(\vec{r} - \vec{r}_i\right) = \sum_n v_{H_n} e^{i\vec{H}_n\cdot\vec{r}}$$

Dynamical Diffraction Theory

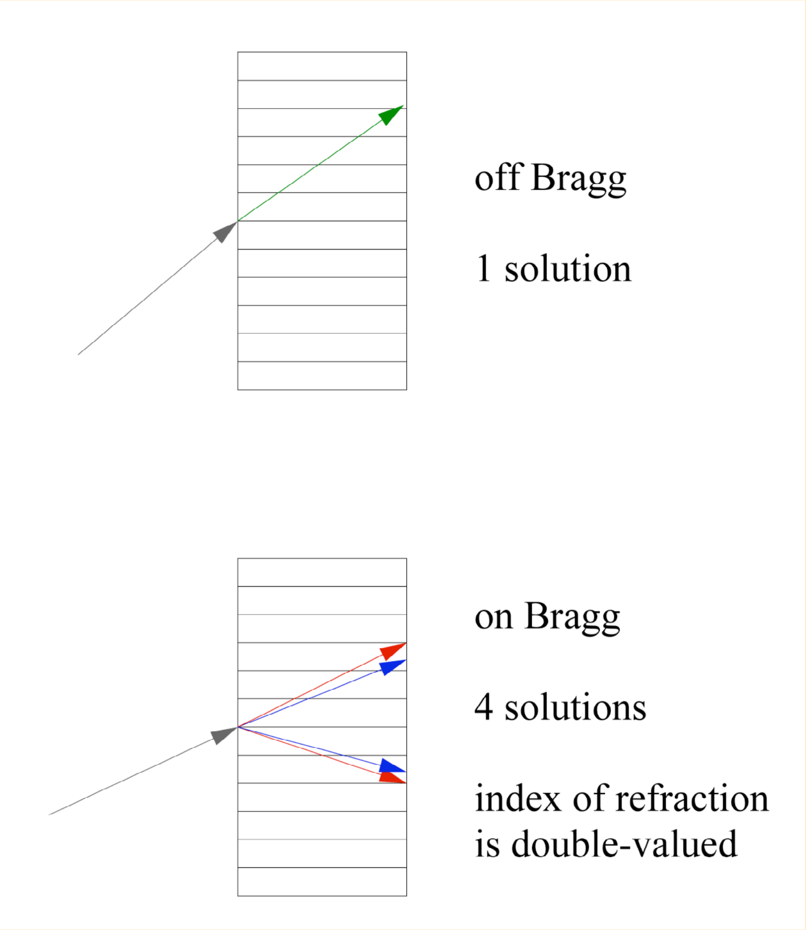


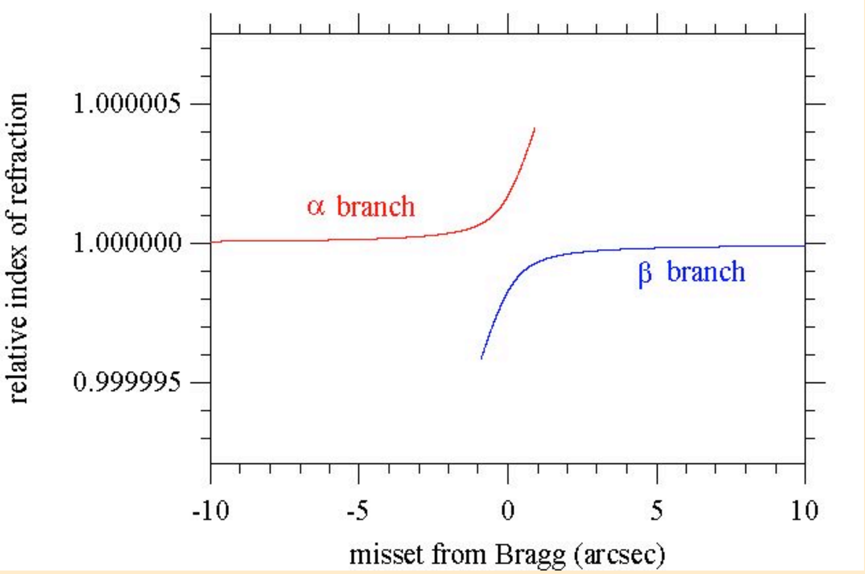
Dispersion Equation: $$\left(K^2 - K_0^2\right)\left(K^2 - K_H^2\right) = v_H^2$$

approximate: $$\left(K - K_0\right)\left(K - K_H\right) = \frac{v_H^2}{4k_0^2}$$

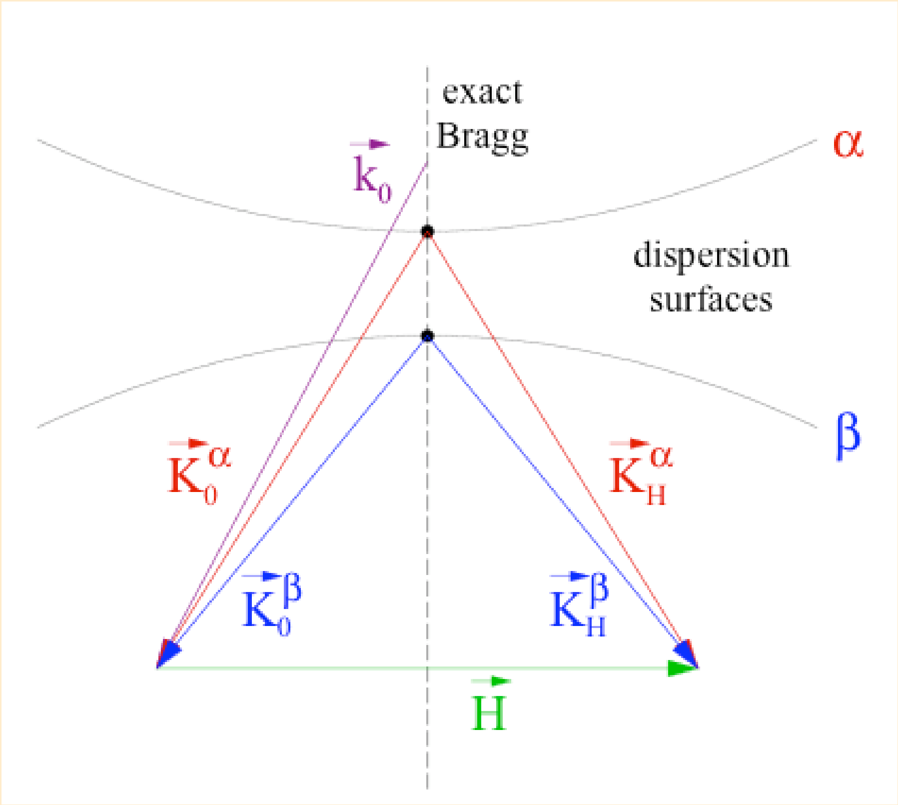
quadratic equation
2 solutions for K_0

Dynamical Diffraction Theory



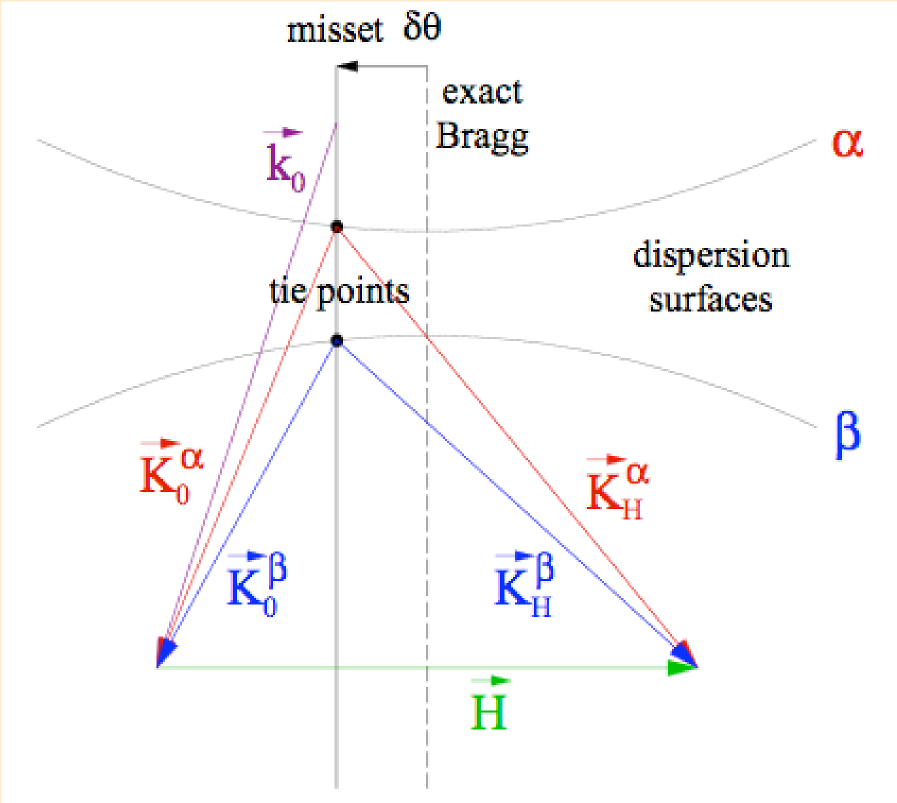


Dynamical Diffraction Theory



internal wave function: $\Psi(\vec{r}) = \psi_0^\alpha e^{i\vec{K}_0^\alpha \cdot \vec{r}} + \psi_0^\beta e^{i\vec{K}_0^\beta \cdot \vec{r}} + \psi_H^\alpha e^{i\vec{K}_H^\alpha \cdot \vec{r}} + \psi_H^\beta e^{i\vec{K}_H^\beta \cdot \vec{r}}$

Dynamical Diffraction Theory



$$\psi_0^\alpha = \frac{1}{2}\left[1 - \frac{y}{\sqrt{1+y^2}}\right] A_0$$

$$\psi_0^\beta = \frac{1}{2}\left[1 + \frac{y}{\sqrt{1+y^2}}\right] A_0$$

$$\psi_H^\alpha = -\frac{1}{2}\left[\frac{1}{\sqrt{1+y^2}}\right] A_0$$

$$\psi_H^\beta = +\frac{1}{2}\left[\frac{1}{\sqrt{1+y^2}}\right] A_0$$

$$y = \frac{k_0 \sin 2\theta_B}{2\nu_H}\delta\theta$$

misset parameter

internal wave function: $\Psi(\vec{r}) = \psi_0^\alpha e^{i\vec{K}_0^\alpha\cdot\vec{r}} + \psi_0^\beta e^{i\vec{K}_0^\beta\cdot\vec{r}} + \psi_H^\alpha e^{i\vec{K}_H^\alpha\cdot\vec{r}} + \psi_H^\beta e^{i\vec{K}_H^\beta\cdot\vec{r}}$

Dynamical Diffraction Theory

Transmitted wave: $$\Psi_{\text{trans}}(\vec{r}) = \psi_{\text{tr}\,0}\, e^{i\vec{k}_0\cdot\vec{r}} + \psi_{\text{tr H}}\, e^{i\vec{k}_H\cdot\vec{r}}$$

$$\psi_{\text{tr}\,0} = \left[\cos\Phi - \frac{iy}{\sqrt{1+y^2}}\sin\Phi\right] e^{i(\phi_1-\phi_0)} A_0$$

$$\psi_{\text{tr H}} = \left[\frac{-iy}{\sqrt{1+y^2}}\sin\Phi\right] e^{-i(\phi_1+\phi_0)} A_0$$

with

$$\phi_0 = \frac{\nu_0 D}{\cos\theta_B} \quad , \quad \phi_1 = \frac{\nu_H D}{\cos\theta_B}$$

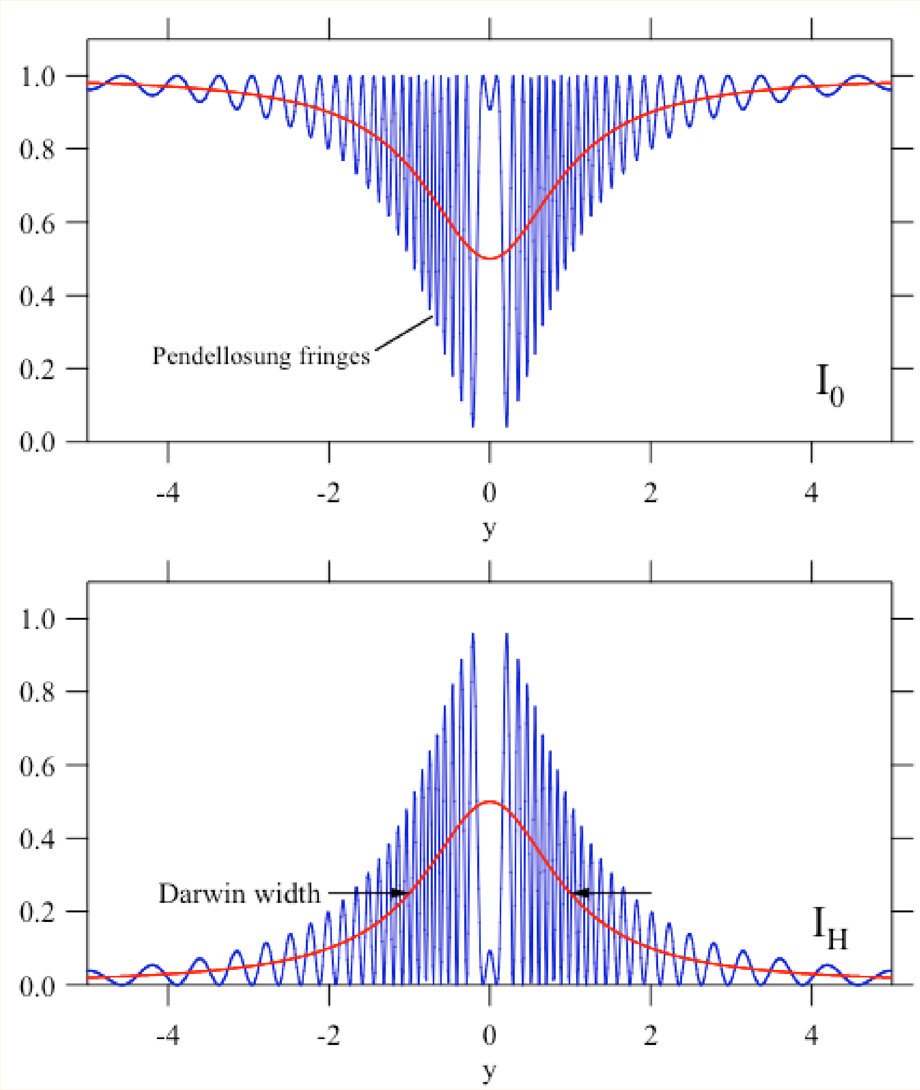
$$\Phi = \left(\nu_H \frac{1}{\sqrt{1+y^2}}\right)\frac{D}{\cos\theta_B}$$

Transmitted intensities:

$$I_0 = \left|\psi_{\text{tr}\,0}\right|^2 = A_0^2\left[\cos^2\Phi + \frac{y^2}{1+y^2}\sin^2\Phi\right]$$

$$I_H = \left|\psi_{\text{tr H}}\right|^2 = A_0^2\left[\frac{1}{1+y^2}\sin^2\Phi\right]$$

Transmitted Intensities



For the ⟨111⟩ reflection in Si at λ=2.70 Å:

$y = 1 \Rightarrow 0.9$ arcsec

Some Consequences of Dynamical Diffraction

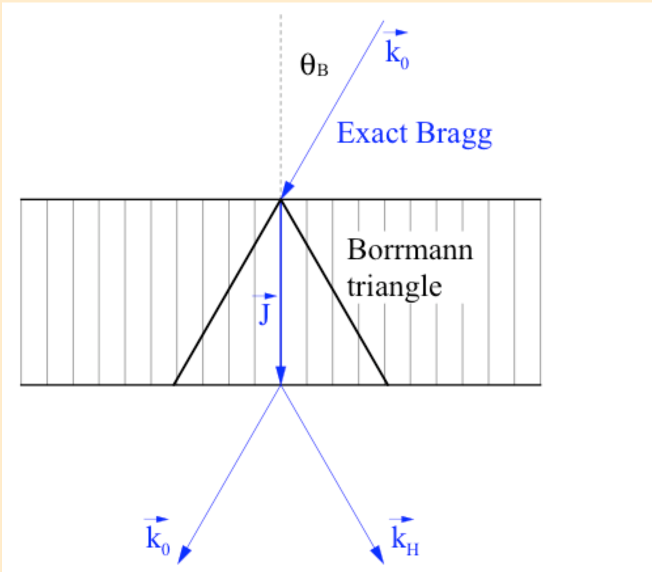
- Pendellösung interference

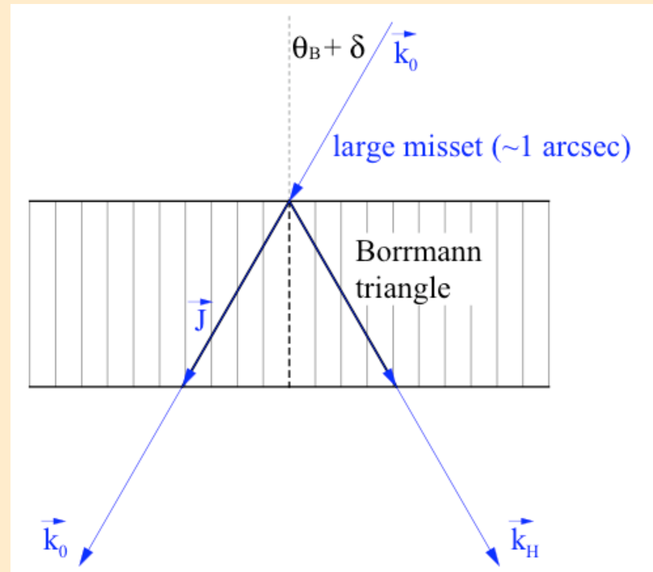
$$\Phi = \left(\nu_H \frac{1}{\sqrt{1+y^2}} \right) \frac{D}{\cos\theta_B}$$

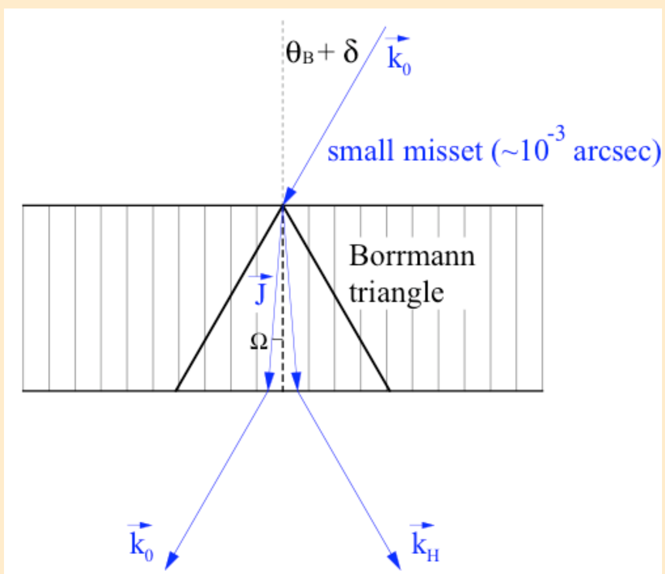
- Anomalous transmission

- Angle amplification

Angle Amplification

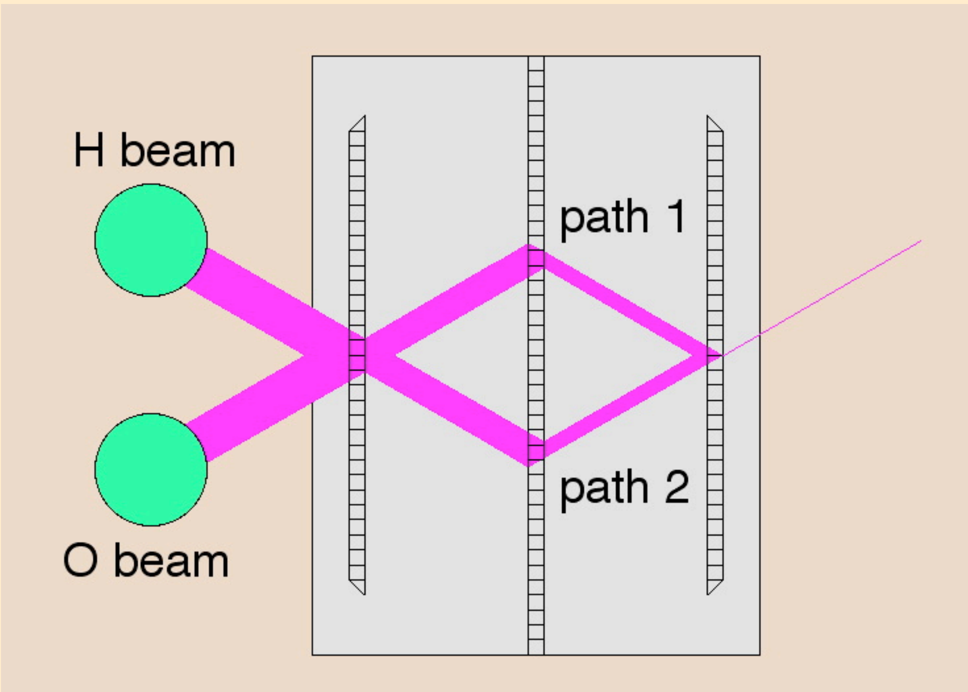


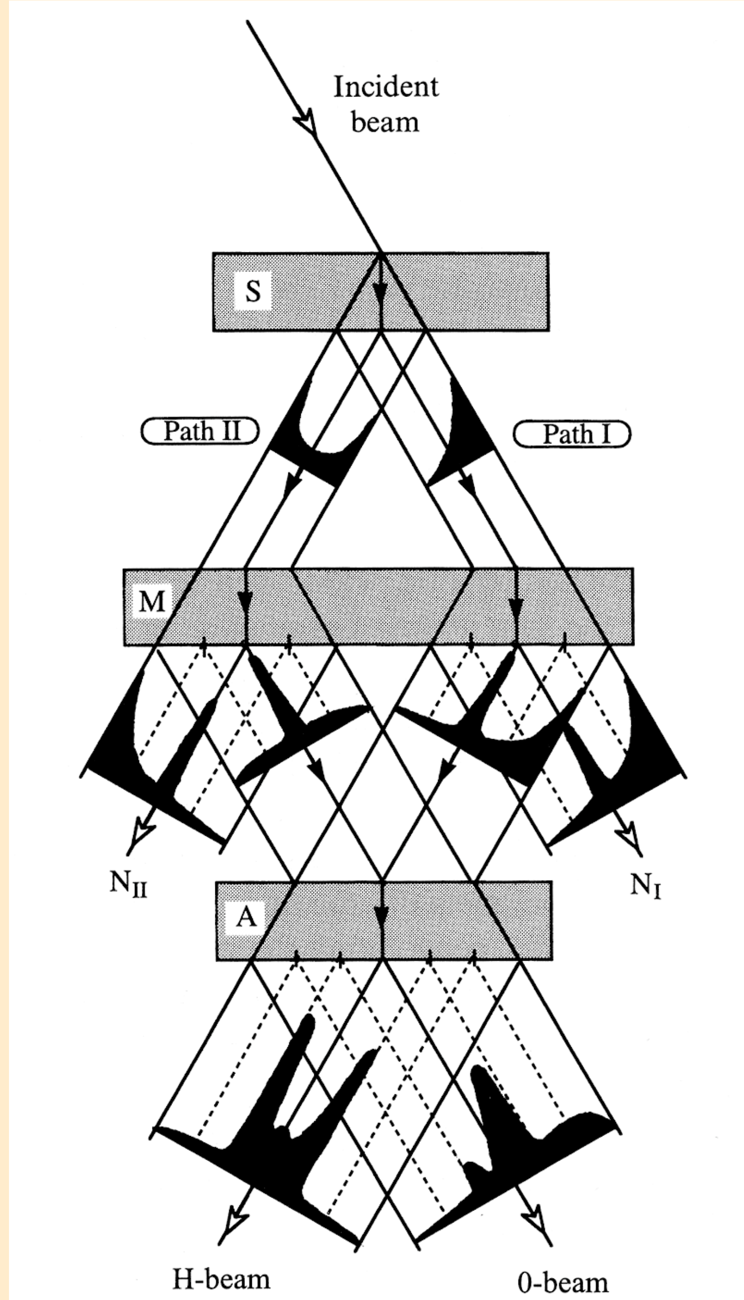




For small δ ($\sim 10^{-3}$ arcsec): $\dfrac{\Omega}{\delta} \approx 10^{6}$

Practical Neutron Interferometer





4π Rotational Symmetry of Spinors

Rotation operator: $R_{\hat{n}}(\alpha) = e^{-\frac{i}{\hbar}\alpha \hat{n}\cdot\vec{S}}$

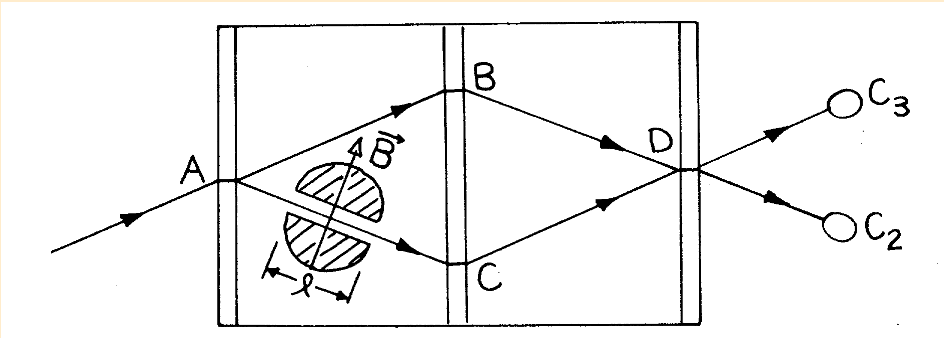
Spin-1/2 particle: $\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$ so $R_{\hat{n}}(\alpha) = e^{-i\frac{\alpha}{2}\hat{n}\cdot\vec{\sigma}}$

Rotations about z-axis: $R_z(\alpha) = \begin{pmatrix} e^{-i\alpha/2} & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix}$

Symmetry:

$$R_z(2\pi)\chi = -\chi$$

$$R_z(4\pi)\chi = \chi$$



Larmor precession phase:

$$\Delta\phi = \pm 2\pi\mu_n m_n \lambda B \ell / \hbar^2$$

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4π-Periodicity of the spinor wave function under space rotation

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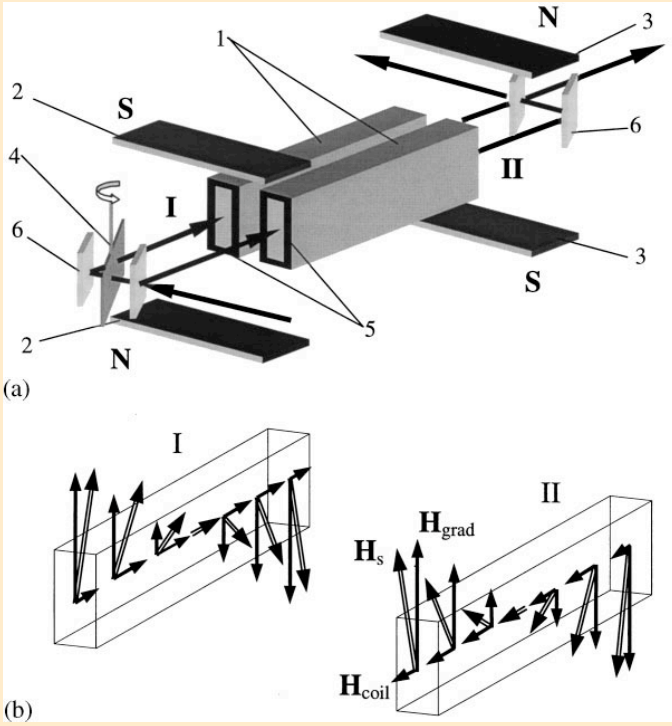
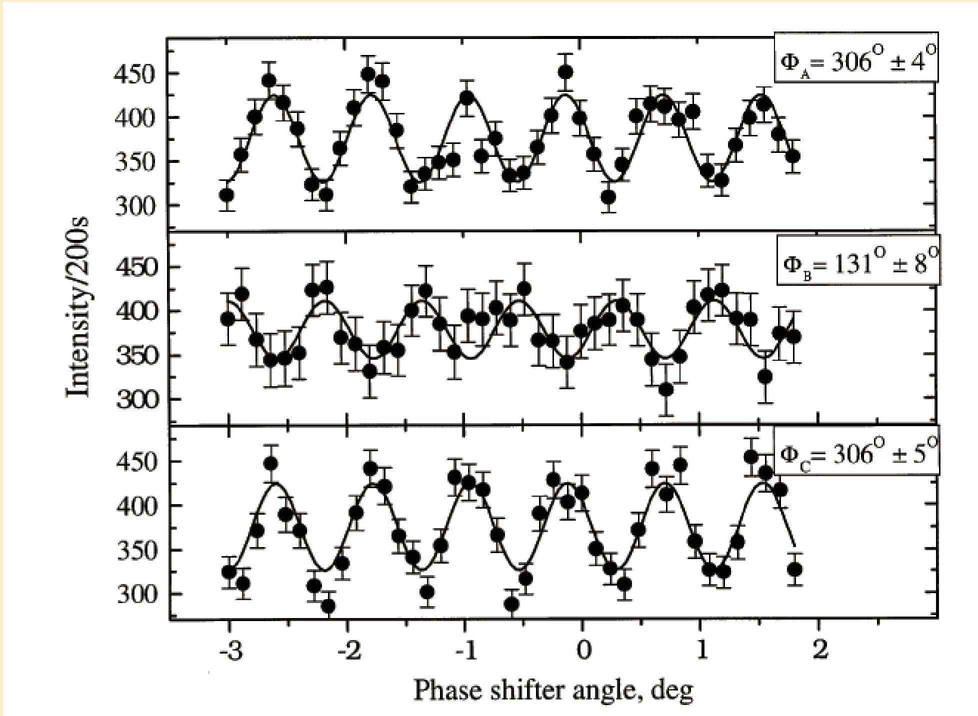
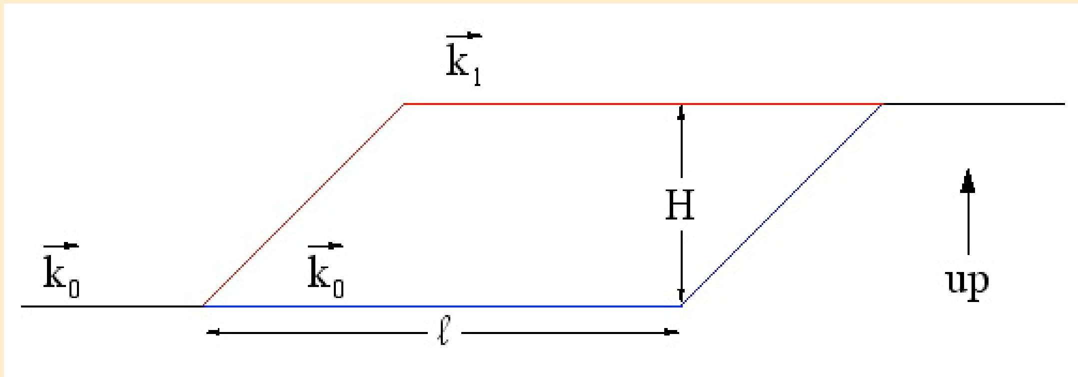




Quantum Phase Shift Due To Gravity (COW Experiments)



$$\Delta\phi = \frac{2\pi\lambda g A}{h^2} m_{\text{in}} m_{\text{grav}}$$

$A = H\ell =$ area of parallelogram

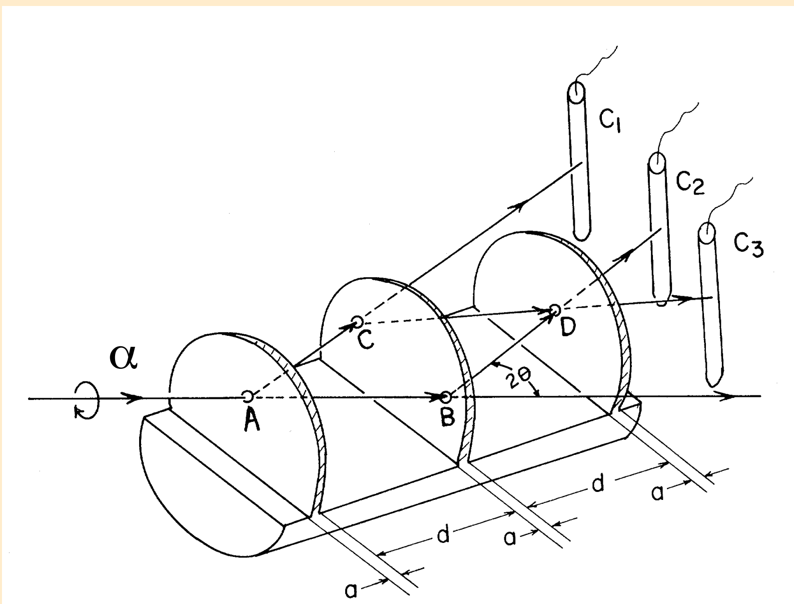
$m_{\text{in}} =$ neutron inertial mass

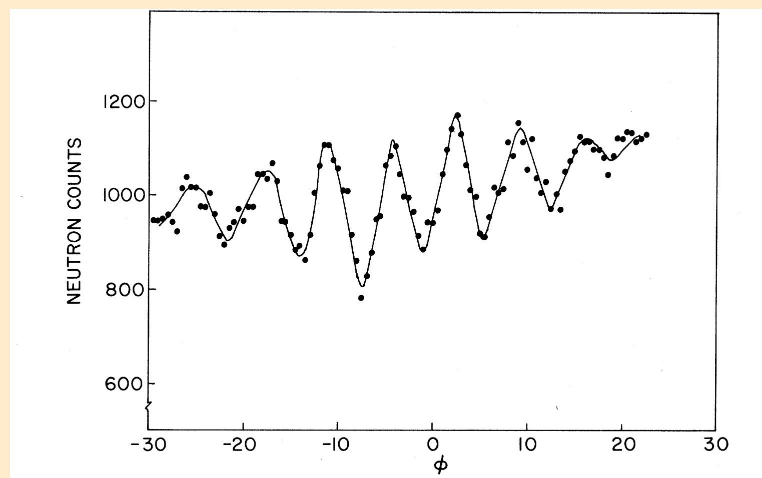
$m_{\text{grav}} =$ neutron gravitational mass

test of weak equivalence principle at the quantum limit

$$\Delta\phi_{\text{grav}} = \frac{2\pi\lambda g A_0}{h^2} m_{\text{in}} m_{\text{grav}} \sin\alpha = q \sin\alpha$$

$A_0 =$ area of parallelogram at $\alpha = 0$





measured: $q = 54.3$
theory: $q = 59.6$

Systematic Effects in the COW Experiments

$$q_{\mathrm{COW}} = \left[\left(q_{\mathrm{grav}} (1 + \varepsilon) + q_{\mathrm{bend}} \right)^2 + q_{\mathrm{Sagnac}}^2 \right]^{\frac{1}{2}}$$

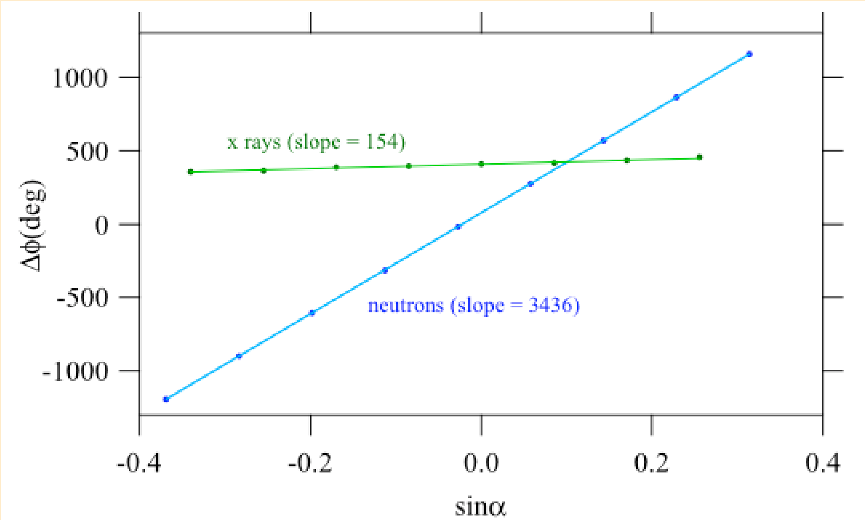
dynamical diffraction correction

bending of interferometer

Earth's rotation

Sagnac effect: $\Delta\phi_{\mathrm{Sagnac}} = \dfrac{2m_{\mathrm{in}}}{\hbar} \vec{\Omega} \cdot \vec{A}$ due to Earth's rotating frame

bending effect: repeat experiment with x rays, different wavelengths



data from Werner, *et al.* (1988)

Littrell, *et al.* (1997) results:

experiment	q_{COW} theory [rad]	q_{COW} meas. [rad]	discrepancy (%)
SS, 440	50. 97(5)	50.18(5)	-1. 6
SS, 220	100. 57(10)	99. 02(10)	-1. 5
LLL, 440	113. 60(10)	112. 62(15)	-0. 9
LLL, 220	223. 80(10)	221. 85(30)	-0. 9

Layer and Greene (1991): x rays do not fill the Borrmann fan as completely as neutrons

Upcoming new effort (H. Kaiser, S. Werner, FEW, et al.):

Suspend interferometer inside chamber filled with $ZnBr_2$+D_2O (floating COW)

Measuring the Neutron's Mean Square Charge Radius Using Neutron Interferometry

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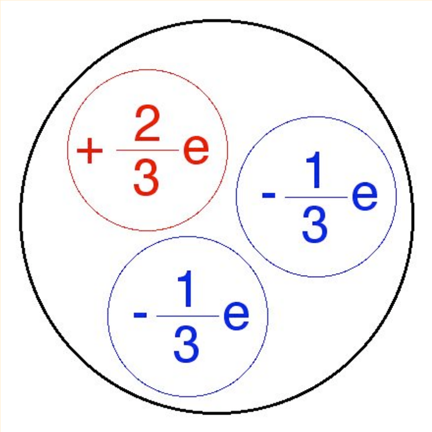
M. Arif, D. L. Jacobson, S. A. Werner
National Institute of Standards and Technology, Gaithersburg, USA

T. C. Black
University of North Carolina, Wilmington, USA

H. Kaiser
Indiana University, Bloomington, USA

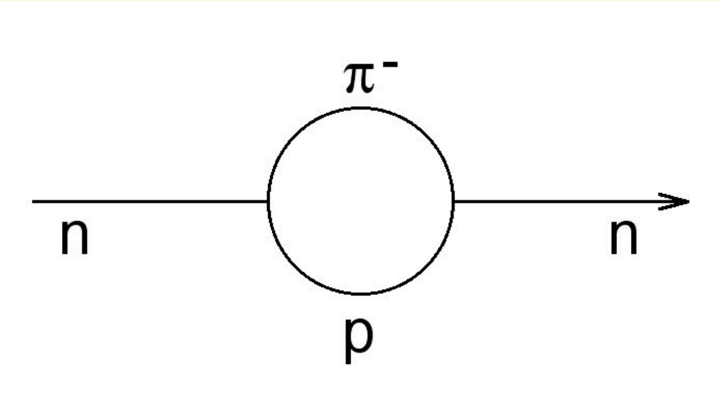
neutron: neutral but consists of charged quarks



neutron mean square charge radius:

$$\langle r_n^2 \rangle = \int \rho(r)\, r^2 d^3 r$$

expected to be negative (positive core, negative skin):



Fermi and Marshall, 1947

Neutron Electric Scattering Form Factor

$G_E^n(Q^2)$ = Fourier transform of neutron charge density (Breit frame)

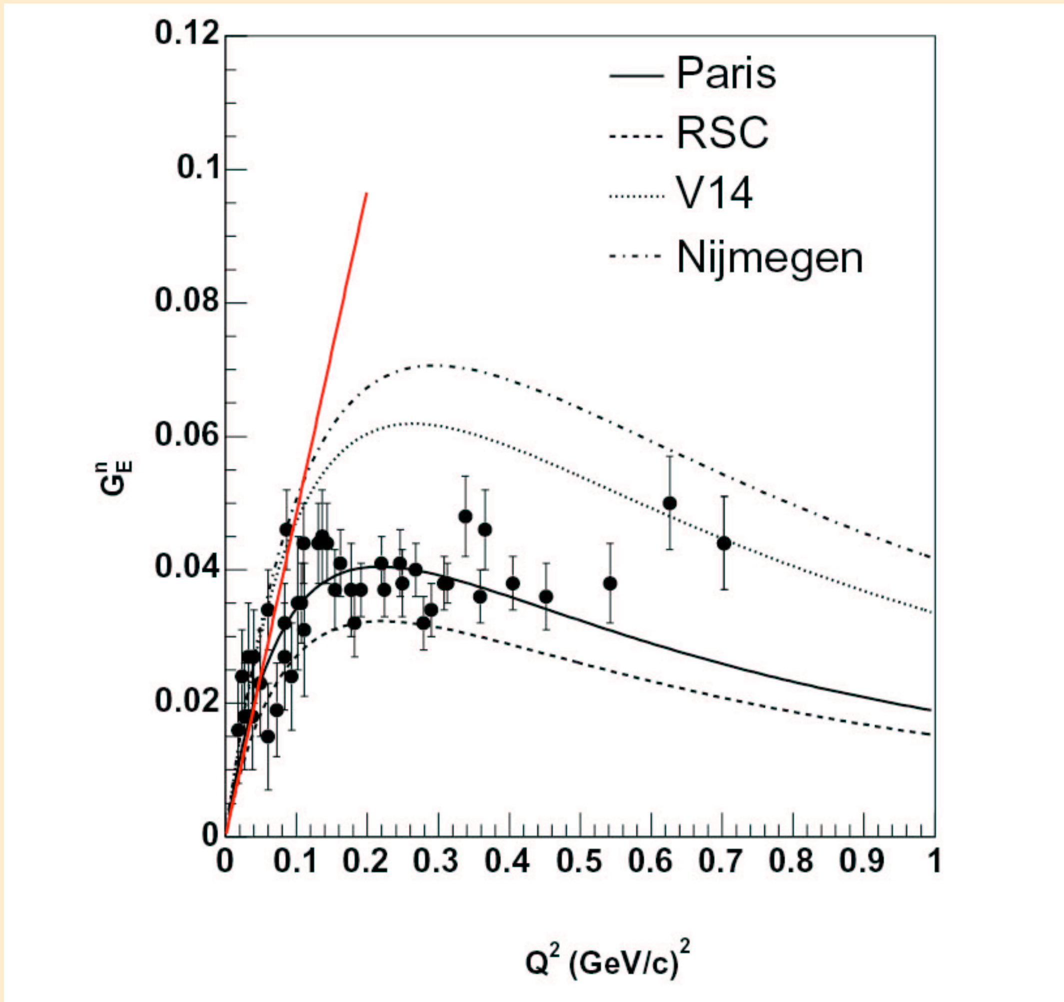
Expanding in momentum transfer Q^2 :

$$G_E^n(Q^2) = q_n - \frac{1}{6}\left\langle r_n^2 \right\rangle Q^2 + \ldots$$

In the low Q^2 limit:

$$\left\langle r_n^2 \right\rangle = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

$\langle r_n^2 \rangle$ constrains the slope of $G_E(Q^2)$ in electron scattering experiments and theory (*e.g*. Bates, Jefferson Lab)



V. Ziskin Ph.D. thesis, 2005

Neutron-Atom Coherent Scattering Length

$$b_{\text{coh}} = b_N + Z\left[1 - f(q)\right]b_{ne}$$

Fourier transform of charge density

$$f(q) = \frac{1}{\sqrt{2\pi}}\int e^{iq\cdot r}\rho_{\text{atom}}(r)\,d^3r$$

b_{ne} = neutron-electron scattering length

In 1st Born approximation: $\left\langle r_n^2 \right\rangle = 3a_0\left(\frac{m_e}{m_n}\right)b_{ne} = \left(86.34\ \text{fm}\right)b_{ne}$

Foldy Scattering Length

$$b_F = -\frac{\gamma e^2}{2m_e c^2} = -1.468 \times 10^{-3} \text{ fm}$$

from neutron's magnetic moment

Incorrect interpretation: $b_{ne}(\text{meas.}) = b_{\text{intrinsic}} + b_F$

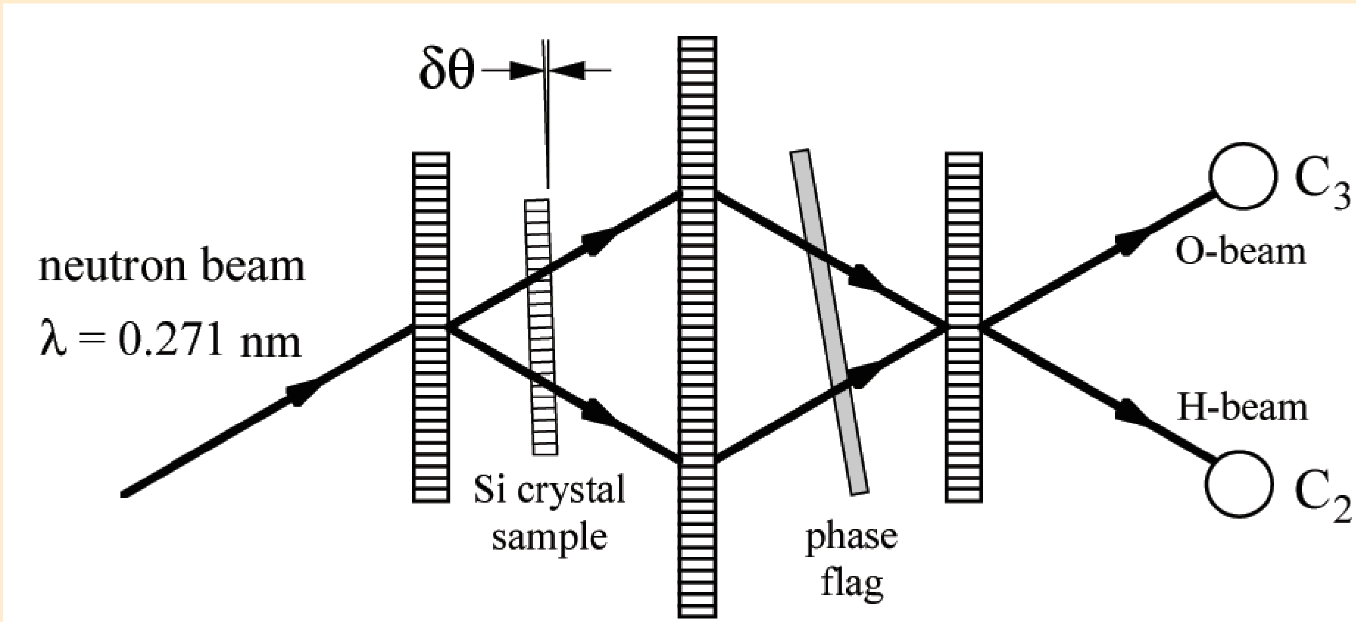
Correct interpretation: The experimentally measured value of b_{ne} is *entirely* due to the static charge distribution in the neutron.
[N. Isgur, Phys. Rev. Lett. **83**, 272 (1999)]

Previous Experiments

TABLE I. Experimental results of b_{ne} in units of 10^{-3} fm.

Experiment	Target	Result	Reference
Angular scattering	Ar	-0.1 ± 1.8	1947 [7] Fermi
Transmission	Bi	-1.9 ± 0.4	1951 [8] Havens
Angular scattering	Kr, Xe	-1.5 ± 0.4	1952 [9] Hamermesh
Mirror reflection	Bi/O	-1.39 ± 0.13	1953 [10] Hughes
Angular scattering	Kr, Xe	-1.4 ± 0.3	1956 [11] Crouch
Crystal spectrometer transmission	Bi	-1.56 ± 0.05	1959 [2] Melkonian
		-1.49 ± 0.05	1976 in Ref. [15]
		$-1.44\pm0.033\pm0.06$	1997 this work
Angular scattering	Ne, Ar, Kr, Xe	-1.34 ± 0.03	1966 [12] Krohn
Angular scattering	Ne, Ar, Kr, Xe	-1.30 ± 0.03	1973 [13] Krohn
Single crystal scattering	^{186}W	-1.60 ± 0.05	1975 [14] Alexandrov
Filter-transmission, mirror reflection	Pb	-1.364 ± 0.025	1976 [15] Koester
Filter-transmission, mirror reflection	Bi	-1.393 ± 0.025	1976 [15] Koester
n-TOF transmission, mirror reflection Ref. [17]	Bi	-1.55 ± 0.11	1986 [16] Alexandrov
Filter-transmission, mirror reflection	Pb, Bi	-1.32 ± 0.04	1986 [17] Koester
n-TOF transmission	thorogenic ^{208}Pb	$-1.31\pm0.03\pm0.04$	1995 [1] Kopecky
		$-1.33\pm0.027\pm0.03$	1997 this work
Filter-transmission, mirror reflection	Pb-isotopes, Bi	-1.32 ± 0.03	1995 [5] Koester
Garching-Argonne compilation	[12,13,15,17]	-1.31 ± 0.03	1986 [3] Sears
Dubna compilation	[14,16]	-1.59 ± 0.04	1989 [19] Alexandrov
Foldy approximation, b_F		-1.468	1952 [18] Foldy

Neutron Interferometer Experiment



off Bragg: $b_{\text{coh}} = b_N + Z[1 - f(0)]b_{ne} = b_N$

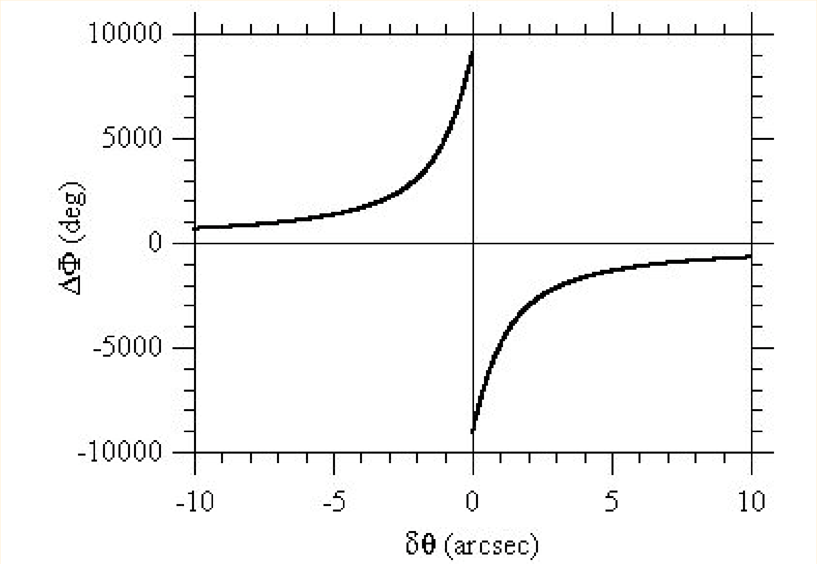
near Bragg: $b_{\text{coh}} = b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$

Dynamical Phase Shift Through Bragg

$$\Delta\Phi_{\text{dyn}} = \frac{\nu_H}{\cos\theta_B}\left(y \pm \sqrt{1+y^2}\right)D$$

D = crystal thickness

scaled misset angle $y = \dfrac{k \sin 2\theta_B}{2\nu_H}$



$$\nu_H = \frac{F_{111}\lambda}{V_{\text{cell}}} = \frac{\sqrt{32}\lambda}{V_{\text{cell}}} b_{\text{coh}}$$

near Bragg: $b_{\text{coh}} = b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$

What we must measure:

1. Net dynamical phase shift through Bragg $\rightarrow$ ν_H $\rightarrow$ $b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$
   to ~10^{-5}

   The maximum slope is ~ 88π/arcsec so we need 0.01 arcsec angular precision to detect every 2 πof phase shift

2. Forward phase shift off Bragg $\rightarrow$ b_N to ~10^{-5} and subtract

3. Neutron wavelength to ~10^{-3}

4. Calculate $f(\vec{H}_{111})$ to ~10^{-3}

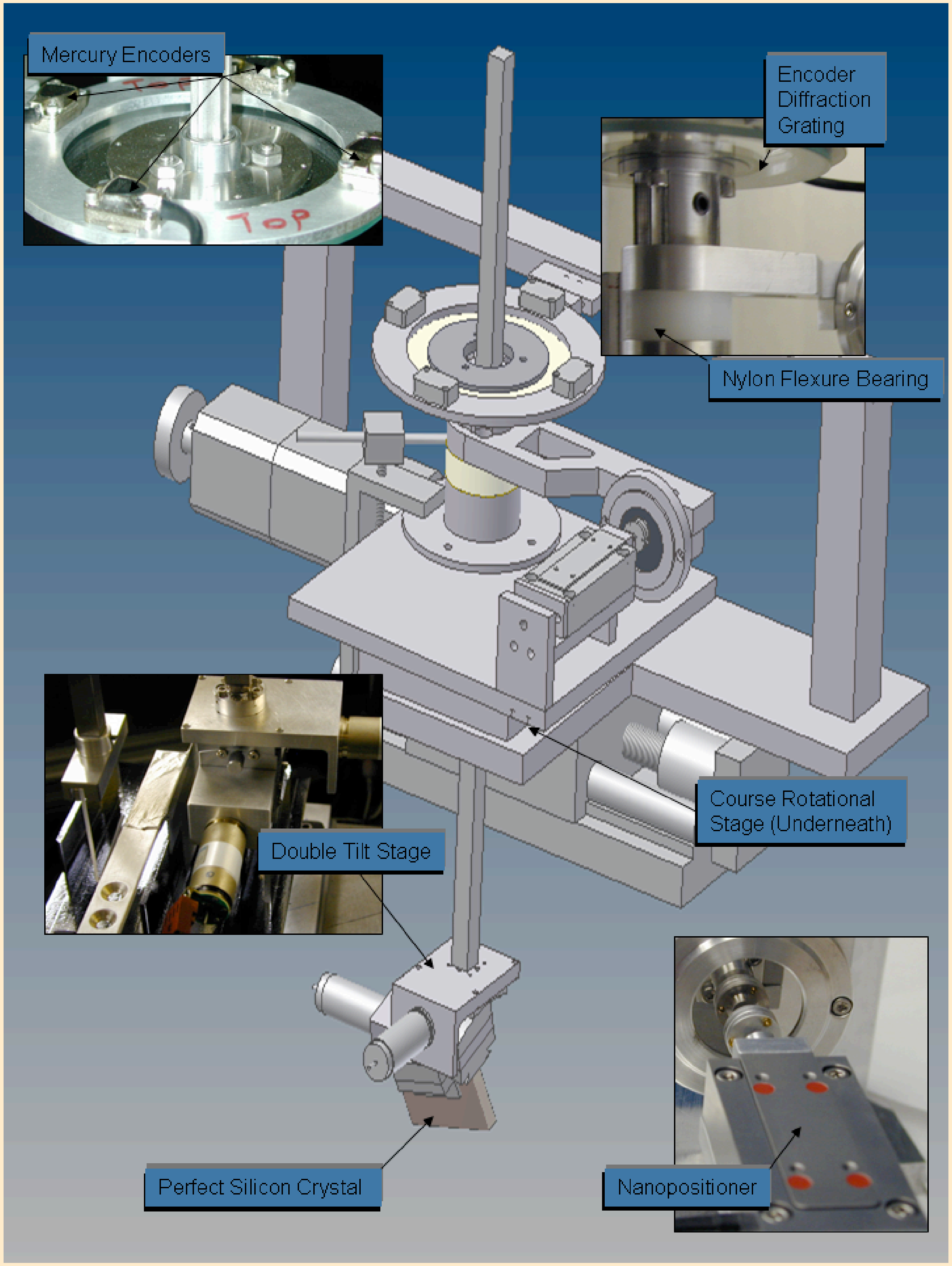
This will give b_{ne}, and hence $\langle r_n^{\,2} \rangle$, to $< 1\%$

Tulane-NIST neutron charge radius experiment

10 cm lever with nylon flexure bearing

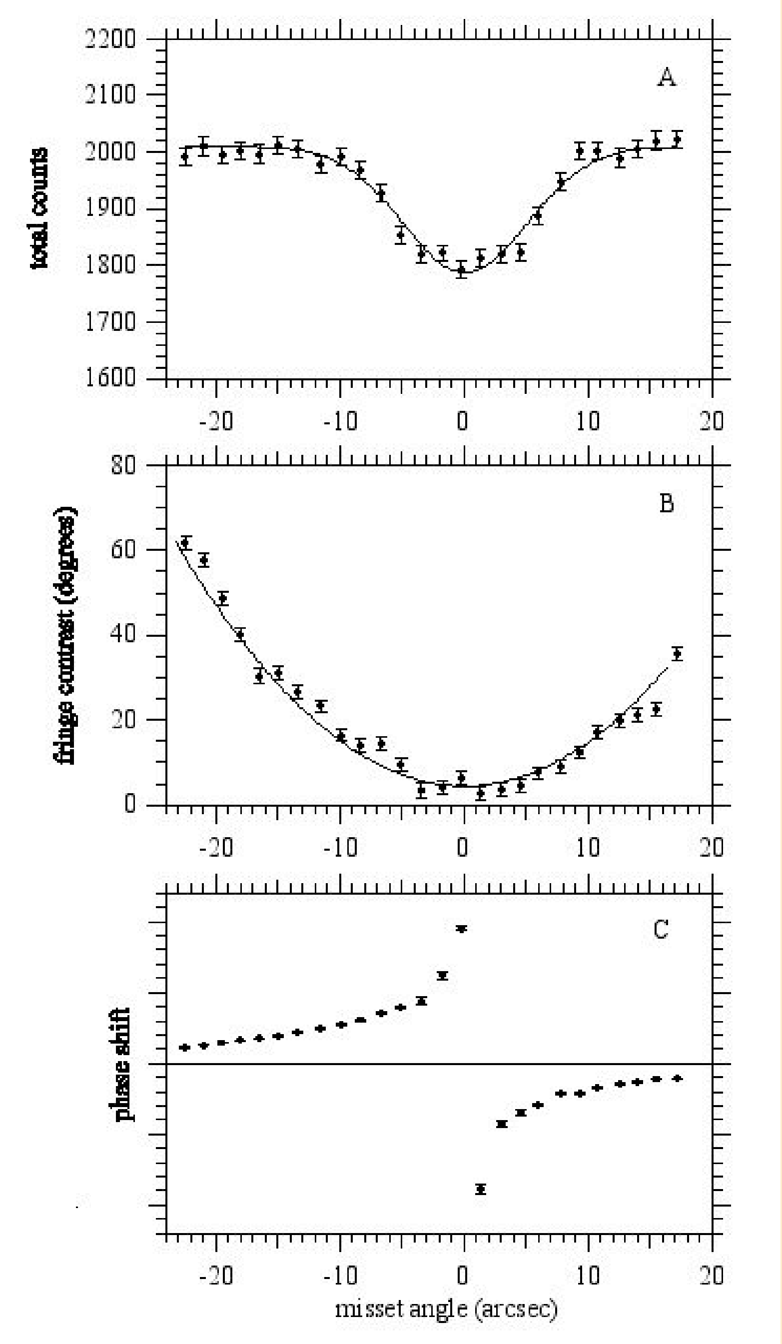
Physik Instrumente
P-753 PZT nanopositioner
25 μm range
1.0 nm precision (.002 arcsec)

Four Micro-E
mercury rotation encoders
.010 arcsec precision



Preliminary Data:

These data were taken at
NIST in September 2005



The Neutron

by Gina Berkeley

When a pion an innocent proton seduces
With neither excuses
Abuses
Nor scorn
For its shameful condition
Without intermission
The proton produces: a neutron is born.
What love have you known
O neutron full grown
As you bombinate into the vacuum alone?
Its spin is 1/2, and its mass is quite large
-about 1 AMU
but it hasn't a charge;
Though it finds satisfaction in strong interaction
It doesn't experience Coulombic attraction
But what can you borrow
Of love, joy, or sorrow
O neutron, when life has so short a tomorrow?

Within its
Twelve minutes
Comes disintegration
Which leaves an electron in mute desolation
And also another ingenuous proton
For other unscrupulous pions to dote on.
At last, a neutrino;
Alas, one can see no
Fulfilment for such a leptonic bambino.
No loving, no sinning
Just spinning and spining
Eight times through the globe without ever beginning...
A cycle mechanic
No anguish or panic
For such is the pattern of life inorganic.
O better
The fret a
Poor human endures
Than the neutron's dichotic
Robotic
Amours.