Theory of Neutron β-Decay

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Inspiration

"It is a part of the adventure of science to try to find a limitation in all directions and to stretch a human imagination as far as possible everywhere. Although at every stage it has looked as if such an activity was absurd and useless, it often turns out at least not to be useless."

Richard P. Feynman,

in "Computing Machines of the Future",

from Feynman and Computation, A. J. G. Hey, ed., 2002

Lecture Plan

1. Preliminaries: Phenomenology

Particles decay weakly via a "V-A" interaction and may violate *C*, *P*, *CP*, and *T*.

2. The Standard Model

A Theory of Nearly Everything, specified by particle content, symmetry, and renormalizability.

3. Beyond the Standard Model

How do we know there is a "Beyond"? Can we observe it in terrestrial experiments? What could characterize it?

Entertainment: How does the neutron lifetime make life possible?

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Preliminaries: Phenomenology

How "Weak" is the Weak Interaction?

We know of four fundamental interactions: electromagnetic, strong, weak, and gravitational.

Let's set gravity aside and consider the others exclusively.

Particles of comparable mass can have very different lifetimes.

$$\pi^+ \to \mu^+ \nu_\mu \qquad [99.98\% \text{ of all } \pi^+ \text{ decays}]\,; \qquad \tau_{\pi^+} \sim 2.6 \cdot 10^{-8}\,\mathrm{s}$$

$$\pi^0 \to 2\gamma \qquad [98.8\% \text{ of all } \pi^0 \text{ decays}]\,; \qquad \tau_{\pi^0} \sim 8.4 \cdot 10^{-17}\,\mathrm{s}.$$

$$\Gamma \propto \tau^{-1} \Longrightarrow \frac{|g_{\rm eff}^{em}|^2}{|g_{\rm eff}^{weak}|^2} \sim 10^8 \Longrightarrow |g_{\rm eff}^{em}| \sim 10^4 |g_{\rm eff}^{weak}|$$

whereas

$$\rho^0 \to \pi^+\pi^- \qquad [\sim 100\% \text{ of all } \rho^0 \text{ decays}]$$

$$\rho^0 \to \mu^+\mu^- \qquad [\sim 4.6 \cdot 10^{-5} \text{ of all } \rho^0 \text{ decays}]$$

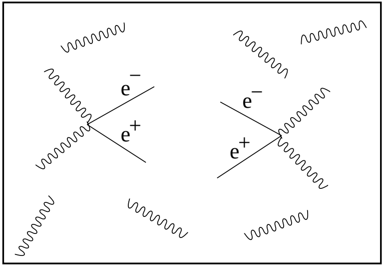
$$\Longrightarrow \frac{|g_{\rm eff}^{em}|^2}{|g_{\rm eff}^{str}|^2} \sim 4 \cdot 10^{-5} \Longrightarrow |g_{\rm eff}^{str}| \sim 10^2 |g_{\rm eff}^{em}|$$

Conclude weak interaction is $\sim 10^6$ times weaker than the strong interaction!

In particle interactions, can we tell...

- Left from Right? (P)
- Positive Charge from Negative Charge? (C)
- Forward in Time from Backward in Time? (T)
- Matter from Antimatter? (CP)

If we "observed" a box of photons at constant temperature $T \sim m_e$, interacting via electromagnetic forces, the answer would be No.



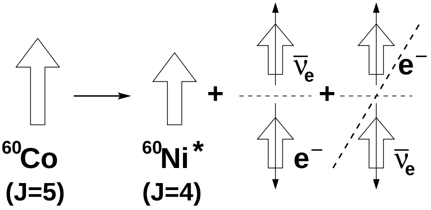
However, ...

The Weak Interactions Violate Parity

There is a "fore-aft" asymmetry in the e^- intensity in $^{60}\vec{Co}$ β-decay....

[Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413 (1957).]

Schematically



$$I_e(\theta) = 1 - \frac{\vec{J}\cdot\vec{p}_e}{E_e}$$

P is violated in the weak interactions!

Both P and C are violated "maximally"

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^+ \to \mu^+ \nu_R) = 0 \quad ; \text{ P violation}$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^- \to \mu^- \overline{\nu}_L) = 0 \quad ; \text{ C violation}$$

The "Two-Component" Neutrino

A Dirac spinor can be formed from two 2-dimensional representations:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

In the Weyl representation for γ^μ,

$$(i\gamma^\mu \partial_\mu - m)\psi = \begin{pmatrix} -m & i(\partial_0 + \sigma \cdot \nabla) \\ i(\partial_0 - \sigma \cdot \nabla) & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

If m=0, ψ_L and ψ_R decouple and are of definite helicity for all p.
Thus, e.g.,

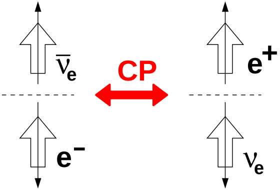
$$i(\partial_0 - \sigma \cdot \nabla)\psi_L(x) \Longrightarrow E\psi_L = -\sigma \cdot p\, \psi_L$$

$$\sigma \cdot \hat{p}\, \psi_L = -\psi_L$$

Note $\bar{\psi} \equiv \psi_L^\dagger \gamma^0$ transforms as a right-handed field.
Experiments $\Longrightarrow$ No "mirror image states": neither $\bar{\nu}_L$ nor ν_R exist.
Possible only if the neutrino is of zero mass.

The Weak Interactions Can Also Violate CP

CP could be a good symmetry even if P and C were violated.
Schematically



$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \overline{\nu}_R) \quad ; \text{ CP invariance!}$$

Weak decays into hadrons, though, can violate *CP*.
There are “short-lived” and “long-lived” K states:

$$K_S \sim \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0) \to \pi^+\pi^- \quad \text{(CP even)}$$

$$K_L \sim \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \to \pi^+\pi^-\pi^0 \quad \text{(CP odd)}$$

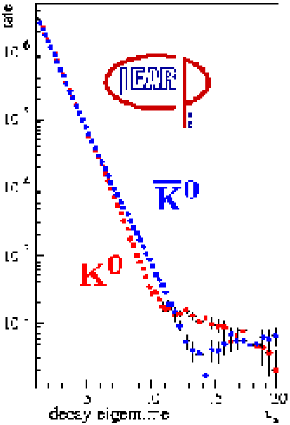
However, $K_L \to 2\pi$ as well! K_S and K_L do not have definite CP!

[Christenson, Cronin, Fitch, Turlay, PRL 13, 138 (1964).]

Matter and Antimatter are Distinguishable

The decay rates for $K^0, \bar{K}^0 \to \pi^+\pi^-$ are appreciably different.

[Thomas Ruf (CPLEAR), http://cplear.web.cern.ch/]



All Observed Interactions Conserve CPT

The CPT Theorem

Any Lorentz-invariant, local quantum field theory in which the observables are represented by Hermitian operators must respect CPT. [Pauli, 1955: Lüders, 1954]

CPT $\Longrightarrow$ the lifetimes, masses, and the absolute values of the magnetic moments of particles and anti-particles are the same!

Note, e.g.,

$$\frac{|M_{K^0} - M_{\bar{K}_0}|}{M_{\rm avg}} < 10^{-18} \text{ @90\% } CL$$

$$\frac{|M_p - M_{\bar{p}}|}{M_{\rm avg}} < 10^{-8} \text{ @90\% } CL$$

Thus CP $\leftrightarrow$ T violation. Tests of CPT and Lorentz invariance are ongoing.

"A search for an annual variation of a daily sidereal modulation of the frequency difference between co-located ^{129}Xe and ^{3}He Zeeman masers sets a stringent limit on boost-dependent Lorentz and CPT violation involving the neutron, consistent with no effect at the level of 150 nHz...." [F. Canè et al., PRL 93 (2004) 230801]

Discrete Symmetries — P, T, and C

Parity P:

Parity reverses the momentum of a particle without flipping its spin.

$$P a_p^s P^\dagger = a_{-p}^s \quad , \quad P b_p^s P^\dagger = -b_{-p}^s \qquad \Longrightarrow P\psi(t,x)P^\dagger = \gamma^0\psi(t,-x)$$

Time-Reversal T:

Time-reversal reverses the momentum of a particle and flips its spin.
It is also antiunitary; note $[x,p] = i\hbar$.

$$T a_p^s T^\dagger = a_{-p}^{-s} \qquad T b_p^s T^\dagger = b_{-p}^{-s} \qquad \Longrightarrow T\psi(t,x)T^\dagger = -\gamma^1\gamma^3\psi(-t,x)$$

Charge-Conjugation C:

Charge conjugation converts a fermion with a given spin into an antifermion with the same spin.

$$C a_p^s C^\dagger = b_p^s \quad , \quad C b_p^s C^\dagger = a_p^s \qquad \Longrightarrow C\psi(t,x)C^\dagger = -i\gamma^2\psi^*(t,x)$$

Transformations of Lorentz Bilinears under P, T, and C

Notation: $\xi^\mu = 1$ for $\mu = 0$ and $\xi^\mu = -1$ for $\mu \neq 0$.
$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad ; \quad \sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

	$\bar{\psi}\psi$	$i\bar{\psi}\gamma_5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma_5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	∂_μ
	S	P	V	A	T	
P	$+1$	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$	$(-1)^\mu$
T	$+1$	-1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$	$-(-1)^\mu$
C	$+1$	$+1$	-1	$+1$	-1	$+1$
CPT	$+1$	$+1$	-1	-1	$+1$	-1

S is for Scalar
P is for Pseudoscalar
V is for Vector
A is for Axial-Vector
T is for Tensor

All scalar fermion bilinears are invariant under CPT.

Symmetries of a Dirac Theory

A Lagrangian must be a Lorentz scalar to guarantee Lorentz-invariant equations of motion. E.g., applying the Euler-Lagrange eqns to

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

yield Dirac equations for ψ and $\bar{\psi}$.
We can form two currents

$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu \psi(x) \quad ; \quad j^{\mu 5}(x) = \bar{\psi}(x)\gamma^\mu \gamma^5 \psi(x)$$

j^μ is always conserved if $\psi(x)$ satisfies the Dirac equation:

$$\partial_\mu j^\mu = (\partial_\mu \bar{\psi})\gamma^\mu \psi + \bar{\psi}\gamma^\mu \partial_\mu \psi = (im\bar{\psi})\psi + \bar{\psi}(-im\psi) = 0\,,$$

whereas $\partial_\mu j^{\mu 5} = 2im\bar{\psi}\gamma^5\psi$ — it is conserved only if $m = 0$.
By Noether's theorem a conserved current follows from an invariance in $\mathcal{L}_{Dirac}$:

$$\psi(x) \to e^{i\alpha}\psi(x) \quad ; \quad \psi(x) \to e^{i\alpha\gamma^5}\psi(x)$$

The last is a chiral invariance; it only emerges if $m = 0$.

Symmetries of a Dirac Theory

To understand why it is a chiral invariance, we note in the $m = 0$ limit that

$$j_L^\mu = \bar{\psi}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right)\psi \quad , \quad j_R^\mu = \bar{\psi}\gamma^\mu \left(\frac{1+\gamma^5}{2}\right)\psi \, .$$

The vector currents of left- and right-handed particles are separately conserved.

Note in Weyl representation

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The factor $(1 \pm \gamma^5)$ acts to project out states of definite handedness.

$$\psi_L \equiv \left(\frac{1-\gamma^5}{2}\right)\psi \quad , \quad \psi_R \equiv \left(\frac{1+\gamma^5}{2}\right)\psi \, .$$

Electromagnetism

We assert that if we couple a Dirac field ψ to an electromagnetic field A^μ
j^μ is the electric current density. ψ can describe a free electron.

$$\psi = u(p)e^{-ip\cdot x} \Longrightarrow (\gamma_\mu p^\mu - m)\psi = 0\,.$$

By "canonical substitution" $p^\mu \to p^\mu + eA^\mu$

$$(\gamma_\mu p^\mu - m)\psi = \gamma^0 V\psi \quad ; \quad \gamma^0 V = -e\gamma_\mu A^\mu$$

In $\mathcal{O}(e)$ the amplitude for an electron scattering from state $i \to f$ is

$$T_{fi} = -i\int \psi_f^\dagger V(x)\psi_i(x)\, d^4x = -i\int j_\mu^{fi} A^\mu\, d^4x \quad \text{with} \quad j_\mu^{fi} = -e\bar{\psi}_f\gamma_\mu\psi_i$$

For $e - p$ scattering, e.g., we have

$$T_{fi} = -i\int j_\mu^e(x)\left(-\frac{1}{q^2}\right) j_\mu^p(x)\, d^4x = -i\mathcal{M}(2\pi)^4\delta^{(4)}(p + k - p' - k')$$

$$\mathcal{M} \equiv -\frac{e^2}{q^2}\left(j_\mu^{em}\right)_p\left(j^{em\,\mu}\right)_e = (e\bar{u}_p(p')\gamma_\mu u_p(p))\left(-\frac{e^2}{q^2}\right)(-e\bar{u}_e(k')\gamma^\mu u_e(k))$$

A current-current interaction.

Fermi Theory

Now consider $n \to p e^- \bar{\nu}_e$.

Fermi's crucial insight was to realize that the weak currents could be modelled after electromagnetism:

$$\mathcal{M} = G(\bar{u}_p(p')\gamma_\mu u_n(p))(\bar{u}_e(k')\gamma^\mu u_\nu(k))$$

The observation of $e-p$ capture suggests

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\left\{(\bar{\psi}_p\gamma_\mu\psi_n)(\bar{\psi}_e\gamma^\mu\psi_\nu) + h.c.\right\}$$

An interaction with charged weak currents.

A weak neutral current was discovered in 1973.

G_F is the Fermi constant, though $G_F \sim 10^{-5}(\text{GeV})^{-2}$.

Suggests the interaction is mediated by massive, spin-one particles.

Fermi's interaction cannot explain the observation of parity violation.

Nor can it explain the $|\Delta J| = 1$ ("Gamow-Teller") transitions observed in nuclear β-decay.

Some $A \times A$ or $T \times T$ interaction has to be present.

Enter the $V - A$ Law....

[Feynman, Gell-Mann, 1958; Sudarshan and Marshak, 1958]

The V-A Law

A "universal" charged, weak current:

$$\mathcal{L} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\left\{\mathcal{J}^\lambda \mathcal{J}_\lambda^\dagger + \mathcal{J}_\lambda^\dagger \mathcal{J}^\lambda\right\} \quad \text{with} \quad \mathcal{J}_\lambda = {j^l}_\lambda + {j^h}_\lambda$$

For the leptons...

$$j^{l\,\lambda} = \bar{\psi}_e \gamma^\lambda (1-\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu (k')\gamma^\lambda (1-\gamma_5)\psi_{\nu_\mu} + \bar{\psi}_\tau (k')\gamma^\lambda (1-\gamma_5)\psi_{\nu_\tau}$$

which describes $\nu_l \to l^-$ and $l^+ \to \bar{\nu}_l$ and asserts the leptons do not mix under the weak interactions.

The "V-A" law is equivalent to a "two-component" neutrino picture.

The interactions of the hadrons (quarks) are much richer.

- The strong interaction is strong!
- The quarks *mix* under the weak interactions. E.g., $K^+ \to \mu^+\nu$ is observed. Recall K^+ is $(u\bar{s})$.

Let us continue to focus on neutron β-decay. Recall n is ddu and p is uud.

Isospin is an approximate symmetry:

$M_n = 939.565\,\text{MeV}$ $M_p = 938.272\,\text{MeV}$ $(M_n - M_p)/M_n \ll 1$.

$n \to pe^-\bar{n}u_e$ occurs because isospin is broken $\Longrightarrow$ large τ_n.

Polarized Neutron β-decay in a V-A Theory

$$d^3\Gamma = \frac{1}{(2\pi)^5 2m_B}\left(\frac{d^3\boldsymbol{p}_p}{2E_p}\frac{d^3\boldsymbol{p}_e}{2E_e}\frac{d^3\boldsymbol{p}_\nu}{2E_\nu}\right)\delta^4(p_n - p_p - p_e - p_\nu)\frac{1}{2}\sum_{spins}|\mathcal{M}|^2$$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\langle p(p_p)|J^\mu(0)|\vec{n}(p_n,P)\rangle[\bar{u}_e(p_e)\gamma_\mu(1-\gamma_5)u_\nu(p_\nu)]$$

$$\langle p(p_p)|J^\mu(0)|\vec{n}(p_n,P)\rangle = \bar{u}_p(p_p)(f_1\gamma^\mu - i\frac{f_2}{M_n}\sigma^{\mu\nu}q_\nu + \frac{f_3}{M_n}q^\mu - g_1\gamma^\mu\gamma_5 + i\frac{g_2}{M_n}\sigma^{\mu\nu}\gamma_5 q_\nu - \frac{g_3}{M_n}\gamma_5 q^\mu)u_{\vec{n}}(p_n,P)$$

Note $q = p_n - p_p$ and for baryons with polarization P,
$u_{\vec{n}}(p_n,P) \equiv (\frac{1+\gamma_5 \not{P}}{2})u_n(p_n)$

f_1 (g_V)	Fermi or Vector	g_1 (g_A)	Gamow-Teller or Axial Vector
f_2 (g_M)	Weak Magnetism	g_2 (g_T)	Induced Tensor or Weak Electricity
f_3 (g_S)	Induced Scalar	g_3 (g_P)	Induced Pseudoscalar

Since $(M_n - M_p)/M_n \ll 1$, a “recoil” expansion is efficacious.
To see how, consider the observables....

Correlation Coefficients

$$d^3\Gamma \propto E_e|\boldsymbol{p}_e|(E_e^{\rm max} - E_e)^2 \times$$
$$[1 + a\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu}{E_e E_\nu} + \boldsymbol{P}\cdot(A\frac{\boldsymbol{p}_e}{E_e} + B\frac{\boldsymbol{p}_\nu}{E_\nu} + D\frac{\boldsymbol{p}_e \times \boldsymbol{p}_\nu}{E_e E_\nu})]dE_e d\Omega_e d\Omega_\nu$$

A and B are P odd, T even, whereas D is (pseudo)T odd, P even.
$\lambda \equiv |g_1/f_1| > 0$ and predictions:

$$a = \frac{1-\lambda^2}{1+3\lambda^2} \qquad A = 2\frac{\lambda(1-\lambda)}{1+3\lambda^2} \qquad B = 2\frac{\lambda(1+\lambda)}{1+3\lambda^2} \qquad [+\mathcal{O}(R)]$$

implying $1 + A - B - a = 0$ and $aB - A - A^2 = 0$, testing the V-A structure of the SM to recoil order, $\mathcal{O}(R)$, $R \sim E_e^{\rm max}/M_n \sim 0.0014$.
Currently

$$a = -0.102 \pm 0.005 \qquad A = -0.1162 \pm 0.0013 \qquad B = 0.983 \pm 0.004$$

so that the relations are satisfied.
With $\tau_n = 885.7 \pm 0.8\,\text{sec}$ and $\tau_n \propto f_1^2 + 3g_1^2$ more tests are possible.

RPP, Particle Data Group, 2002.

Symmetries of the Hadronic, Weak Current

The values of the 6 couplings (assuming T invariance) are constrained by symmetry.

- Conserved-Vector Current ("CVC") Hypothesis
- Absence of Second-Class Currents ("SCC")
- Partially Conserved Axial Current ("PCAC") Hypothesis

CVC:
The charged weak current and isovector electromagnetic current form an **isospin triplet.** [Feynman and Gell-Mann, 1958]

$$J_\mu^{em,q} = \frac{2}{3}\bar{\psi}_u \gamma^\mu \psi_u - \frac{1}{3}\bar{\psi}_d \gamma^\mu \psi_d$$

$$J_\mu^{em,q} = e_0 \bar{\psi}_q \gamma^\mu I \psi_q + e_1 \bar{\psi}_q \gamma^\mu \tau_3 \psi_q \quad \text{with} \quad \psi_q = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

$$\tau_3 \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} = \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} \quad ; \quad \tau_3 \begin{pmatrix} 0 \\ \psi_d \end{pmatrix} = -\begin{pmatrix} 0 \\ \psi_d \end{pmatrix} \quad ; \quad e_{0,1} = \frac{1}{2}(e_u \pm e_d)$$

Symmetries of the Hadronic, Weak Current

Thus

$$J_\mu^{em\,N} = \bar{\psi}[F_1^S(q^2)\gamma^\mu - i\frac{F_2^S(q^2)}{M_n}\sigma^{\mu\nu}q_\nu + \frac{F_3^S(q^2)}{M_n}q^\mu]e_0 I\psi$$
$$+\bar{\psi}[F_1^V(q^2)\gamma^\mu - i\frac{F_2^V(q^2)}{M_n}\sigma^{\mu\nu}q_\nu + \frac{F_3^V(q^2)}{M_n}q^\mu]e_1\tau_3\psi$$

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad \text{and} \quad \tau_+ \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} = \begin{pmatrix} \psi_p \\ 0 \end{pmatrix}$$

The CVC hypothesis implies

$$f_1(q^2) = F_1^V(q^2) \quad \text{and} \quad f_1(q^2) \to 1 \quad \text{as } q^2 \to 0$$
$$f_2(q^2) = F_2^V(q^2)$$
$$f_3(q^2) = F_3^V(q^2) = 0 \quad \text{(current conservation)}$$

$f_1(0) = (1 + \Delta_R^V)V_{ud}$ Δ_R^V starts in $\mathcal{O}(\alpha)$!
[tested to $\mathcal{O}(0.3\%)$ in $0^+ \to 0^+$ decays]
$f_2(0)/f_1(0) = (\kappa_p - \kappa_n)/2 \approx 1.8529$
[tested to $\mathcal{O}(10\%)$ in $A = 12$ system]
The Ademollo-Gatto theorem makes the second test more interesting.

Symmetries of the Hadronic, Weak Current

SCC: "Wrong" G-parity interactions do not appear if isospin is an exact symmetry.
$G \equiv C \exp(i\pi T_2)$ where T_2 is a rotation about the 2-axis in isospin space.

$$\exp(i\pi T_2)\psi = -i\tau_2\psi = \begin{pmatrix} -\psi_n \\ \psi_p \end{pmatrix}$$

$$GV_\mu^{(I)}G^\dagger = +V_\mu^{(I)} \quad ; \quad GA_\mu^{(I)}G^\dagger = -A_\mu^{(I)} \quad \text{“first class”}$$
$$GV_\mu^{(II)}G^\dagger = -V_\mu^{(II)} \quad ; \quad GA_\mu^{(II)}G^\dagger = +A_\mu^{(II)} \quad \text{“second class”}$$

no SCC: $g_2 = 0$ and $f_3 = 0$
(tested to $\mathcal{O}(10\%)$ in $A = 12$ system (combined CVC/SCC test))
PCAC: g_1/f_1 is set by strong-interaction physics:
Goldberger-Treiman relation $\frac{g_1(0)}{f_1(0)} = g_{\pi NN}\frac{f_\pi}{M_N}$
Can test some of these relationships through experiments sensitive to recoil-order effects.

Correlation Coefficients in Recoil Order

Consider a and A in recoil order for CVC test. [cf. CVC test in mass 12]
Define $x = \frac{E_l}{E_l^{max}}$ $[0 \leq x \leq 1]$, $\epsilon = (\frac{M_e}{M_n})^2$,
and $R = \frac{E_l^{max}}{m_B} = \frac{M_n^2 + M_e^2 - M_p^2}{2M_n^2} \sim 0.0014$ (note $\frac{\epsilon}{R} \sim 2.2 \cdot 10^{-4}$) to yield (here $\lambda \equiv g_1/f_1$ and $\tilde{f}_2 \equiv f_2(0)/f_1(0)$, e.g.)

$$
\begin{aligned}
a = \frac{1-\lambda^2}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\Bigg\{ \frac{\epsilon}{Rx}\Big[& (1-\lambda^2)(1+2\lambda+\lambda^2 \\
& +2\lambda\tilde{g}_2 + 4\lambda\tilde{f}_2 - 2\tilde{f}_3)\Big] + 4R\Big[(1+\lambda^2)(\lambda^2+\lambda \\
& +2\lambda(\tilde{f}_2+\tilde{g}_2))\Big] - Rx\Big[3(1+3\lambda^2)^2 + 8\lambda(1+\lambda^2) \\
& \times(1+2\tilde{f}_2) + 3(\lambda^2-1)^2\beta^2\cos^2\theta\Big]\Bigg\} + \mathcal{O}(R^2,\epsilon)
\end{aligned}
$$

Correlation Coefficients in Recoil Order

$$A = \frac{2\lambda(1-\lambda)}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\left\{ \frac{\epsilon}{Rx}\Big[4\lambda^2(1-\lambda)(1+\lambda + 2\tilde{f}_2) + 4\lambda(1-\lambda)(\lambda\tilde{g}_2 - \tilde{f}_3)\Big] + R\Big[\frac{2}{3}(1+\lambda + 2(\tilde{f}_2+\tilde{g}_2))(3\lambda^2+2\lambda-1)\Big] + Rx\Big[\frac{2}{3}(1+\lambda+2\tilde{f}_2) \times(1-5\lambda-9\lambda^2-3\lambda^3) + \frac{4}{3}\tilde{g}_2(1+\lambda+3\lambda^2+3\lambda^3)\Big]\right\} + \mathcal{O}(R^2,\epsilon) \, .$$

[Gardner, Zhang, 2001; Bilen'kii et al., 1960; Holstein, 1974]

Coefficients of Rx in A and a yield independent determinations of f_2 and g_2.

[Gardner, Zhang 2001]

Were a and A both measured to $\mathcal{O}(0.1)\%$ (using Rx terms), then $\delta\tilde{f}_2$ is 2.5% and $\delta\tilde{g}_2$ is roughly $0.22\lambda/2$, yielding errors comparable to the mass 12 test

cf. $A = 12$ result $-0.02\lambda \leq 2\tilde{g}_2 \leq 0.31\lambda$ at 90% C.L. (CVC) [Minamisono et al., 2002]

Uses axial charge difference (th.) $\Delta y = 0.10 \pm 0.05$!

Beyond "V-A" in Neutron β-Decay

The search for non-V-A interactions continues...

$$
\begin{aligned}
\mathcal{H}_{int} &= (\bar{\psi}_p \psi_n)(C_S \bar{\psi}_e \psi_\nu + C'_S \bar{\psi}_e \gamma_5 \psi_\nu) + (\bar{\psi}_p \gamma_\mu \psi_n)(C_V \bar{\psi}_e \gamma^\mu \psi_\nu + C'_V \bar{\psi}_e \gamma^\mu \gamma_5 \psi_\nu) \\
&-(\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \bar{\psi}_e \gamma^\mu \gamma_5 \psi_\nu + C'_A \bar{\psi}_e \gamma^\mu \psi_\nu) + (\bar{\psi}_p \gamma_5 \gamma_\mu \psi_n)(C_P \bar{\psi}_e \gamma_5 \psi_\nu + C'_P \bar{\psi}_e \psi_\nu) \\
&+\frac{1}{2}(\bar{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \bar{\psi}_e \sigma^{\lambda\mu} \psi_\nu + C'_T \bar{\psi}_e \sigma^{\lambda\mu} \gamma_5 \psi_\nu) + h.c.
\end{aligned}
$$

[Lee and Yang, 1956; note also Gamow and Teller, 1936]

C'_X denote parity-nonconserving interactions.

In polarized neutron (nuclear) β-decay one more correlation appears: b

$$
d^3\Gamma = \frac{1}{(2\pi)^5} \xi E_e |\boldsymbol{p}_e| (E_e^{\max} - E_e)^2 \times
$$

$$
[1 + a\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} + \boldsymbol{P} \cdot (A\frac{\boldsymbol{p}_e}{E_e} + B\frac{\boldsymbol{p}_\nu}{E_\nu} + D\frac{\boldsymbol{p}_e \times \boldsymbol{p}_\nu}{E_e E_\nu})] dE_e d\Omega_e d\Omega_\nu
$$

[Jackson, Treiman, and Wyld, Phys. Rev. 106, 517 (1957)]

Note, e.g.,

$$
b\xi = \pm 2\mathrm{Re}[C_S C_V^* + C'_S C'^*_V + 3(C_T C_A^* + C'_T C'^*_A)]
$$

If the electron polarization is also detected, more correlations enter.

Limits from Nuclear β-Decay

Recent limits on b come from nuclear β-decay:

$b = -0.0027 \pm 0.0029$
from survey of $0^+ \to 0^+$ ("superallowed" Fermi) transitions in nuclei

[Towner and Hardy, J. Phys. G, 2003]

$\tilde{a} \equiv a/(1 + bm_e/\langle E_e \rangle) = 0.9981 \pm 0.0030 \pm 0.0037$
from $0^+ \to 0^+$ pure Fermi decay of ${}^{38m}K$

[A. Gorelov et al. PRL 94, 142501 (2005)]

Both limits are consistent with the Standard Model.

Nuclear β-decay spin-isospin selection rules are dictated by the form of the nonrelativistic transition operator.

$$\sum_{j=1}^{A} \tau_\pm(j) = T_\pm \quad \text{"Fermi"} \Longrightarrow J_f = J_i, T_f = T_i \neq 0$$

$$\sum_{j=1}^{A} \sigma(j) \cdot \tau_\pm(j) \quad \text{"Gamow-Teller"} \Longrightarrow \Delta J = 0, 1 \; (J_i = J_f \neq 0) \,,$$

$$\Delta T = 0, 1 \; (T_i = T_f \neq 0)$$

The Standard Model

Particle Content

The Standard Model contains three generations of quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

with masses ranging from a few MeV to $\sim$ 172 GeV. The s,c,b,t quarks have additional flavor quantum numbers which are preserved by the strong interaction.

It has three generations of leptons

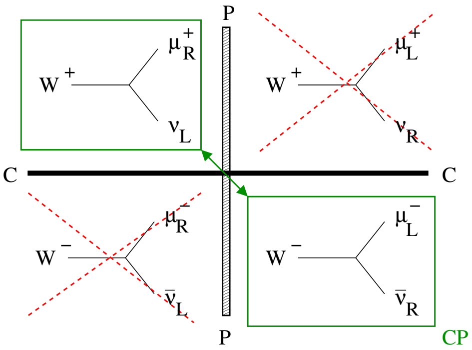
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

with masses ranging from 0 to $\sim$ 1.8 GeV. The leptons do not mix.

It contains gauge bosons: the photon, the gluon, and a triplet of massive, spin-one particles — $W^\pm$ (mass $\sim$ 80 GeV) and Z^0 (mass $\sim$ 91 GeV) with masses generated via a Higgs mechanism. The H^0 is not yet found, though its mass is constrained by direct and indirect searches.

Symmetries of the Standard Model

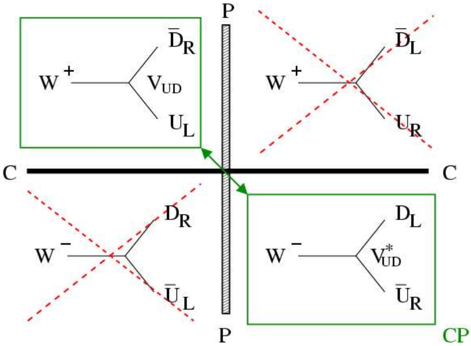
The Standard Model is a quantum field theory with a local $SU(3) \times SU(2) \times U(1)$ gauge symmetry.



In these decays CP is unbroken.

Symmetries of the Standard Model

CP violation does appears naturally in the Standard Model. For quark decays, we have ($U \in (u, c, t)$, $D \in (d, s, b)$)



V_{UD} is an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. If it is complex, then CP violation may occur.
Other mechanisms of CP violation are possible....

The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

The decay $K^- \to \mu^- \bar{\nu}_\mu$ occurs: the quark mass eigenstates *mix* under the weak interactions. By convention

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{\rm weak} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm mass} \quad ; \quad V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

In the Wolfenstein parametrization (1983)

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where $\lambda \equiv |V_{us}| \simeq 0.22$ and is thus “small”. A, ρ, η are real.

Why Study CP Violation?

We live in a Universe of matter.
Confronting the observed abundance of the light elements (^{2}H, ^{4}He, ^{7}Li) with big-bang nucleosynthesis yields

$$\eta = \frac{n_{\rm baryon}}{n_{\rm photon}} = (5.21 \pm 0.5) \times 10^{-10}\ (95\%{\rm CL})$$

This reflects the excess of baryons over anti-baryons when the Universe was a (putative) 100 seconds old.
Why else do we think this?

- The composition of cosmic rays, note $\overline{p}/p \sim 10^{-4}$.
- No evidence for diffuse γ's from $p\overline{p}$ annihilation....

How can this be? Enter CP Violation (Sakharov, 1967).
Estimates of the baryon excess in the Standard Model are much too small, $\eta < 10^{-26}$!! A puzzle.

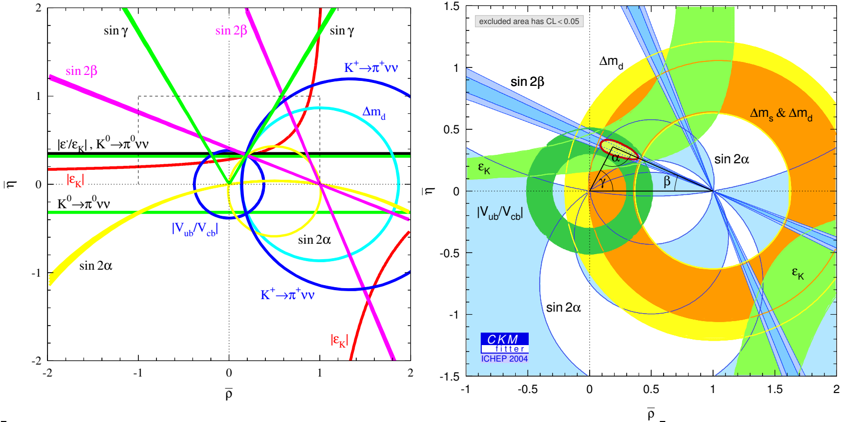
Testing the Quark Mixing Matrix

In the Standard Model (SM)

- There are three “generations” of particles. Thus, the CKM matrix is unitary.
- The unitarity of the CKM matrix and the structure of the weak currents implies that four parameters capture the CKM matrix.
- A real, orthogonal 3×3 matrix is captured by three parameters. The fourth parameter (η) must make $V_{\rm CKM}$ complex.
- All CP-violating phenomena are encoded in η.

To test the SM picture of CP violation we must test the relationships it entails.

Testing the Standard Model



[CKMfitter: hep-ph/0104062, hep-ph/0406184 ; http://ckmfitter.in2p3.fr – August, 2004 update]

How well does the Standard Model work?

[C. Kolda, J. Erler, A. Czarneki, contributions to CIPANP 2006]

Fits to precision electroweak measurements:

Global Fit: [J. Erler, 2005]

$M_H = 88^{34}_{-26}$ GeV

$m_t = 172.5 \pm 2.3$ GeV

$\alpha_s(M_Z) = 0.1216 \pm 0.0017$

$\chi^2/\mathrm{dof} = 47.2/42$ (26%)

indirect only: [J. Erler, 2005]

$m_t = 172.2^{10}_{-7.4}$ GeV

$m_t = 175.6 \pm 3.3$ GeV ($M_H = 117$ GeV fixed)

μ g-2:

$(g-2)_\mu = 116591811(71) \cdot 10^{-11}$

experiment - theory (SM) = $269(95) \cdot 10^{-11}$

differs at the 2.8σ level.

Difficulties with the Standard Model

The Standard Model works unreasonably well, but possesses many arbitrary and/or fine-tuned features.
Moreover,

- it does not include gravity (by design)
- it does not explain dark matter, dark energy
- it cannot explain the baryon asymmetry of the Universe
- it does not explain the number of generations nor the large range of fermion masses
- it does not explain the weak mass scale
- it has a strong CP problem

Beyond The Standard Model

New physics can be explored directly in collider experiments. At low energies the existence of new physics is probed indirectly; it would be inferred from the failure of robust Standard Model predictions.

> Indirect tests cannot reveal the specific nature of new physics, only its existence.

Some Basic Questions

- How do we know there is a “Beyond”?
- Why do we think there is new physics at the TeV scale?
- Why do we think we can probe TeV-scale physics in precision, low-energy experiments?

How do we know there is a "Beyond"?

The "hierarchy problem" (one problem among many) suggests that the Standard Model is incomplete.

We have, however, direct empirical evidence for physics beyond the Standard Model.

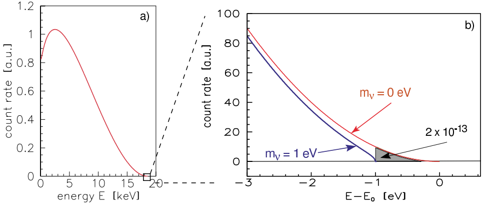
Empirical evidence for neutrino oscillations allows us to conclude $\Delta m^2 \equiv m_i^2 - m_j^2 \neq 0$ with surety.

That is, neutrinos have mass.

We see then that the particle content of the Standard Model is incomplete: there is a ν_R, which is "sterile" under Standard Model interactions.

This is not to say that the effects of neutrino mass are large.

Distortions in the shape of the electron energy spectrum in ^{3}H β-decay near its endpoint bound m_ν^2. [KATRIN, loi]



The Emergence of Physics Beyond the Standard Model

Why do we think there is new physics at $\sim$ 1 TeV?

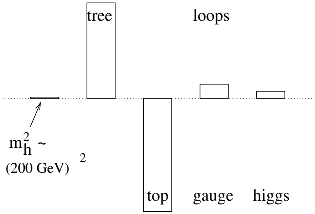
[Schmaltz, hep-ph/0210415]

Suppose we assume the Standard Model is valid for scales $E \leq \Lambda$, where $\Lambda \sim \mathcal{O}(1\text{TeV})$.

At one-loop level, we find large corrections to the tree-level Higgs mass m_{tree}.

All contributions must sum to $m_H^2 \sim (200\text{GeV})^2$, but each one $\sim \Lambda^2$!

At $\Lambda = 10$ TeV, m_{tree} must be tuned to one part in 100!



New physics at the TeV scale can enter to make the cancellations "natural."

Can we probe TeV-scale physics at low energies?

Yes. Let's illustrate this in a **toy model**.

Consider the Gerasimov-Drell-Hearn sum rule:

[Gerasimov, 1966; Drell and Hearn, 1966.]

$$\frac{2\pi\alpha\kappa_i^2}{M^2} = \frac{1}{\pi}\int_0^\infty \frac{\Delta\sigma_i}{\omega}\, d\omega \equiv \frac{1}{\pi}\int_0^\infty \frac{(\sigma_{\mathrm{P}i}(\omega) - \sigma_{\mathrm{A}i}(\omega))}{\omega}\, d\omega$$

The photon and nucleon spins are aligned parallel (P) or anti-parallel (A).

A *linearized* sum rule also exists:

[Holstein, Pascalutsa, and Vanderhaeghen, 2005.]

$$\frac{4\pi\alpha\delta\kappa_i}{M^2} = \frac{1}{\pi}\int_0^\infty \frac{\Delta\tilde{\sigma}_i}{\omega}\, d\omega$$

where $\Delta\tilde{\sigma} \equiv \partial\Delta\sigma/\partial\kappa_{0i}|_{\kappa_{0p}=\kappa_{0n}=0}$. We can compute the contribution to κ_i from

$$\mathcal{L}_{\pi NN} = \frac{g}{2M}\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi\partial_\mu\pi^a$$

We thus determine the loop contribution to κ_i from a "pion" produced at some inelastic threshold ω_{New} in $\gamma - p$ scattering.

Can we probe TeV-scale physics at low energies?

As $\mu \equiv M_\pi / M \to \infty$ this yields

$$\delta\kappa_p = \frac{g^2}{(4\pi)^2}(5 - 4\ln\mu)\frac{1}{\mu^2} + \mathcal{O}(\mu^{-4})$$
$$\delta\kappa_n = \frac{g^2}{(4\pi)^2}2(3 - 4\ln\mu)\frac{1}{\mu^2} + \mathcal{O}(\mu^{-4})$$

Thus if we choose $M_\pi \sim 1$ TeV, $\mu \sim 10^3$, with $g^2/4\pi = 13.5$,

$$\delta\kappa_p = -2.4 \cdot 10^{-5}$$
$$\delta\kappa_n = -5.3 \cdot 10^{-5}$$

The effects of putative TeV-scale physics on the anom. mag. moments are appreciable.
The empirical anomalous magnetic moments are already sufficiently well-known to be impacted by TeV-scale physics, though these effects are obscured by non-perturbative QCD effects.
A challenge to lattice QCD!

The Neutron Lifetime and Big-Bang Nucleosynthesis

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